MICRO-470

EPFL

ELECTRO MAGNETIC scaling in MEMS

H. Shea

v. 2024

Electromagnetics

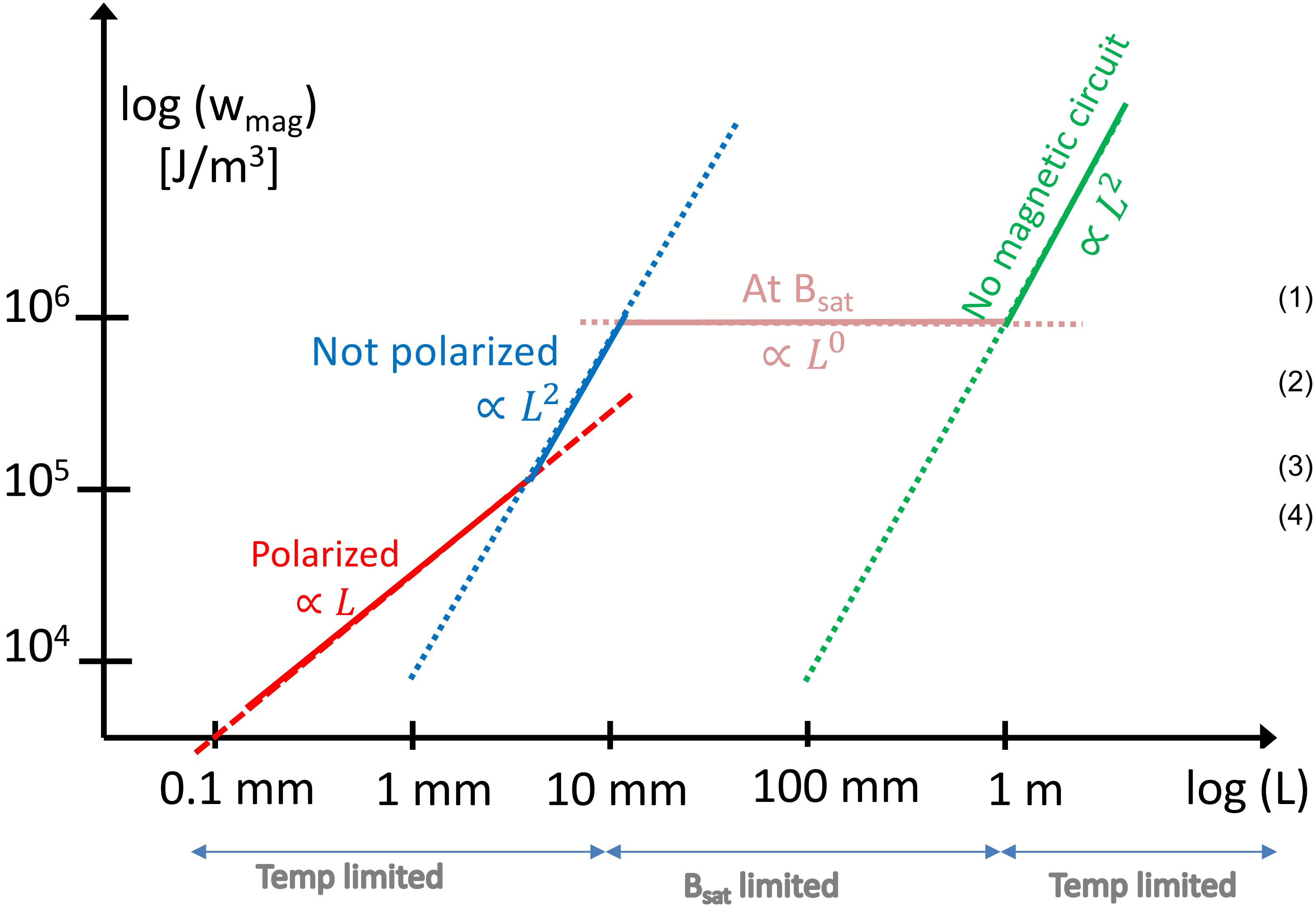
Table of contents

Scaling:
- we can “cheat” scaling laws by using external permanent magnets…
- or we scale only the magnet, or only the coil…

Key concepts on EM scaling

- Types of magnets: diamagnet, paramagnet, ferromagnet, super-paramagnet
- Maximum current in a wire
- Scaling of magnets: transition to super-paramagnetism.
- Magnetic forces
  - Scaling of forces on beads (from wire and from magnet)
  - Scaling of force wire-wire
  - Scaling of force wire-magnet
    - Examples where MEMS is the wire
    - Example where MEMS is the magnet
- Simplistic EM motor:
  - Scaling of energy density and power for the different cases: Polarized vs. non-polarized, case when current allows reaching saturation, and when not.

Summary of Scaling of power density in electromagnetic actuators

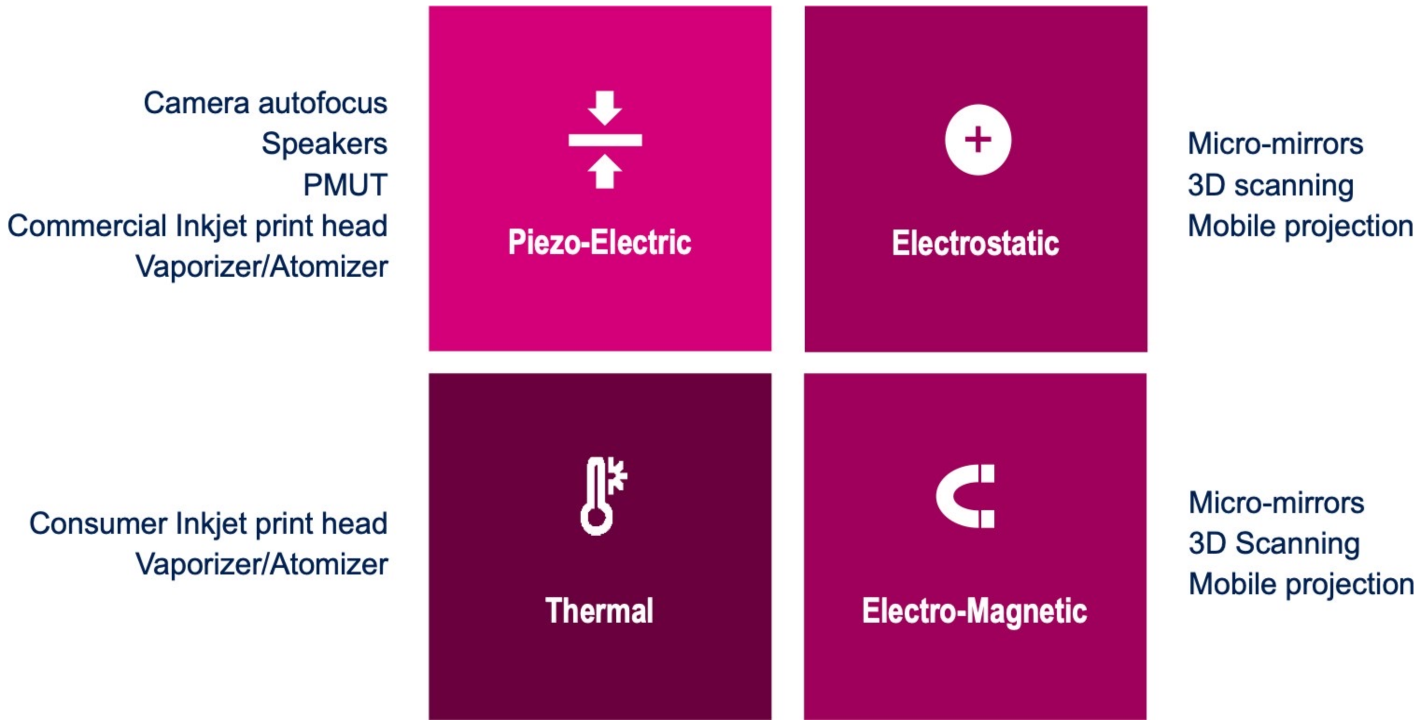


(1) polarized, thermally limited (~L) at very small scale,

(2) not polarized, thermally limited (~L^2) at small scale,

(3) B_{sat} limitation at medium scale,

(4) no magnetic circuit, thermally limited (~L^2) at very large scale



ES was 2D (if we ignore fringing fields)
EM is always 3D (no magnetic monopole!)

EM1. Fundamentals of magnetism

Fundamentals of magnetism

M: magnetization of material (A/m).

m: net magnetic dipole moment per volume = **M** / V

H: magnetic field strength, magnetic flux density (A/m)

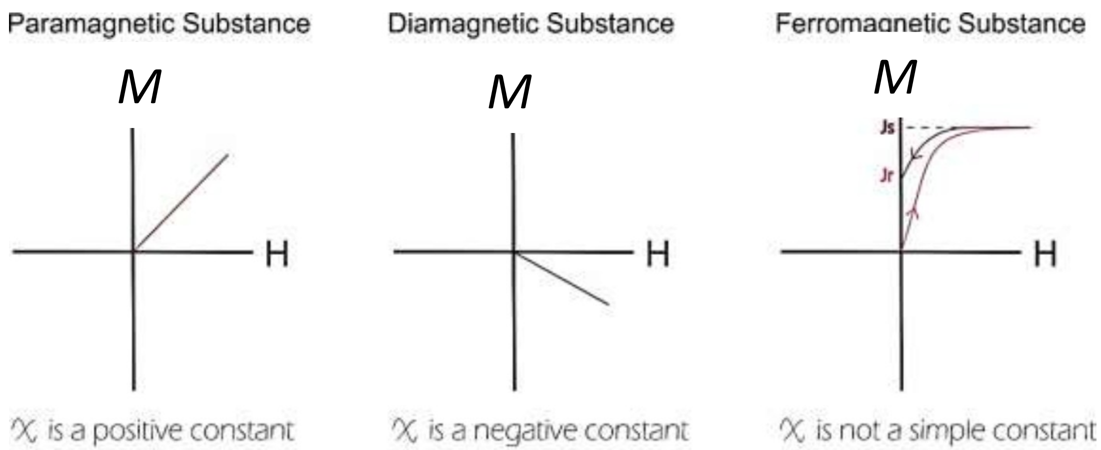
B: magnetic induction or fluc density (or magnetic field strength) (T)

μ: magnetic permeability *χ*: magnetic susceptibility

$$B = \mu H \qquad M = \chi_v H$$

$$B = \mu_0 (H + M) = \mu_0 (1 + \chi_V) H = \mu H$$

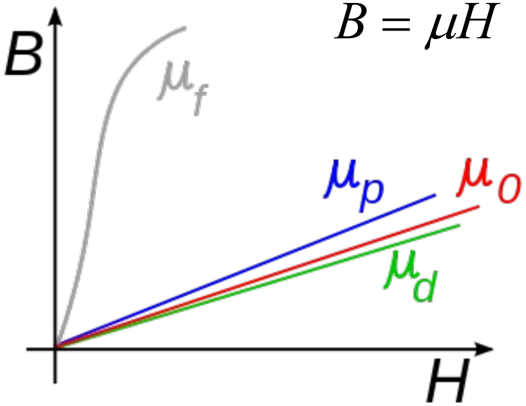
$$\mu = \mu_0 (1 + \chi_v)$$



diamagnetic	*paramagnetic*	*ferromagnetic*	*superparamagnetic*
$\chi_v < 0$ magnetic susceptibility	$\chi_v > 0$	$\chi_v \gg 0$ *and below T_{curie}*	$\chi_v \gg 0$ *and very small size so thermal fluctuations matter (small ferromagnet)*
$\chi_v \sim -10^{-5}$	$\chi_v \sim +10^{-5}$	$\chi_v \sim 10^2$ to 10^6	$\chi_v \sim 10^{-5}$ to 10^6
Water, graphite  20T	*Al, W, Na, wood, etc…*	*- soft (low coercivity, used for core) or* *- hard (high coercivity, used for magnet)*	*Small powder of ferromagnets typically*

http://www.ru.nl/hfml/research/levitation/diamagnetic/

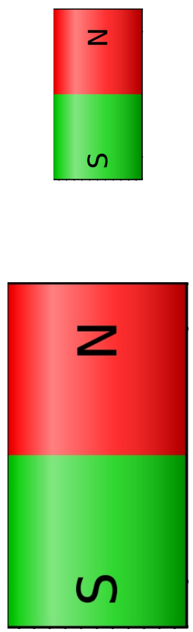
$$B = \mu H \qquad \mu = \mu_0 (1 + \chi_v) \qquad M = \chi_v H$$



https://en.wikipedia.org/wiki/Permeability_%28electromagnetism%29

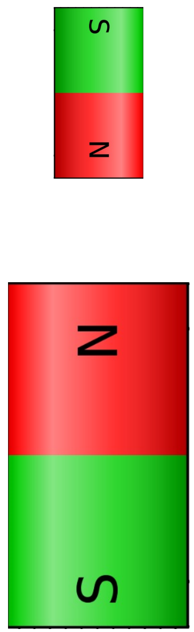
Energy density $w = \dfrac{BH}{2} = \dfrac{\mu H^2}{2} = \dfrac{B^2}{2\mu}$

Paramagnetic or soft ferromagnetic object magnetization lines up with magnet)

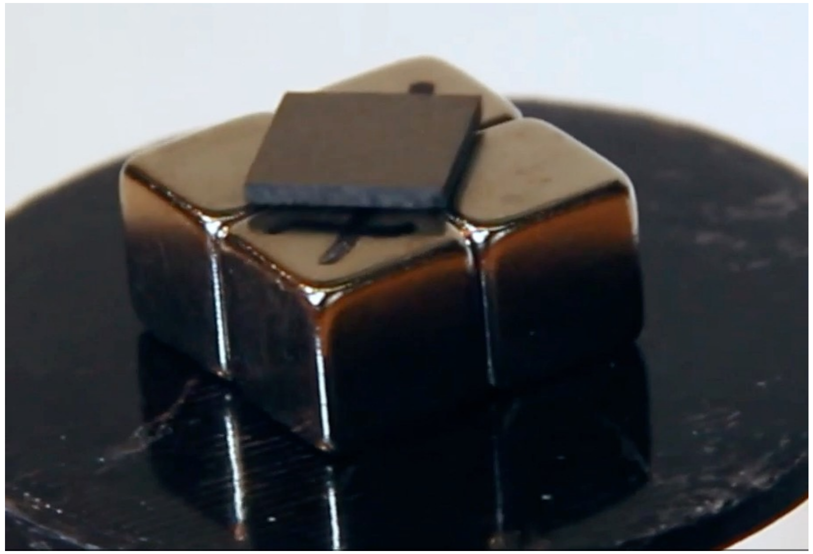


magnet

Diamagnetic object magnetization is opposite to the magnet

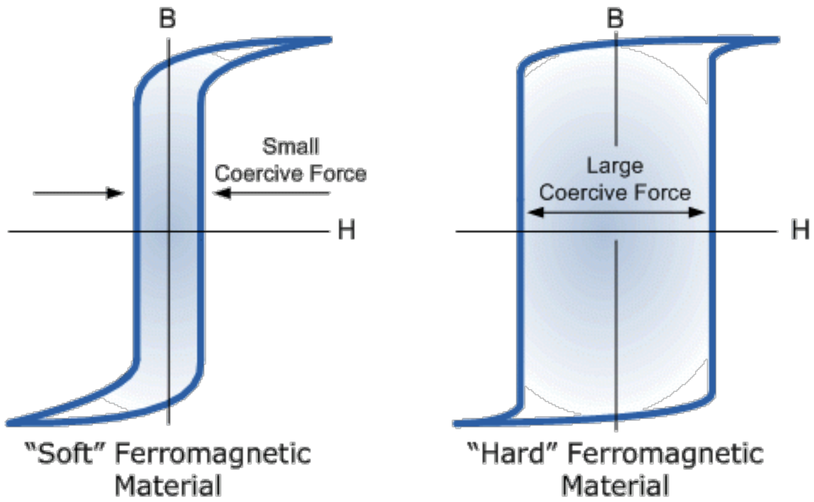


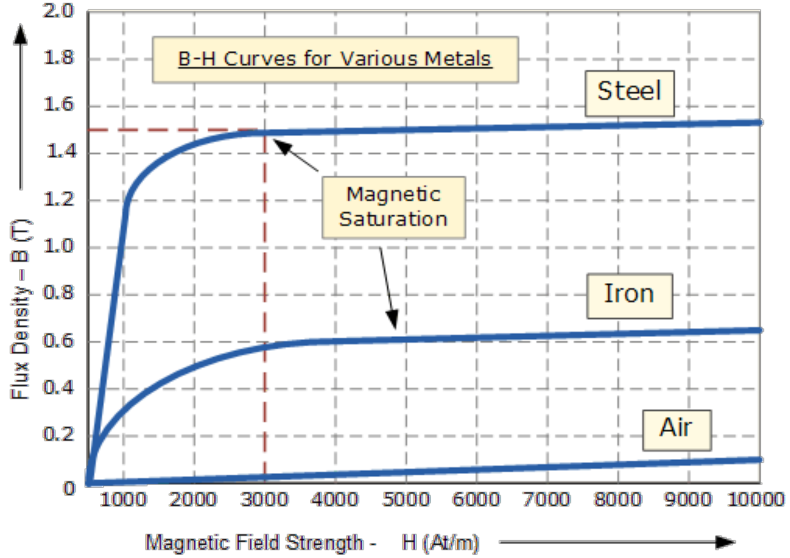
magnet



$$F = m\nabla \vec{B}$$

Ferromagnetism



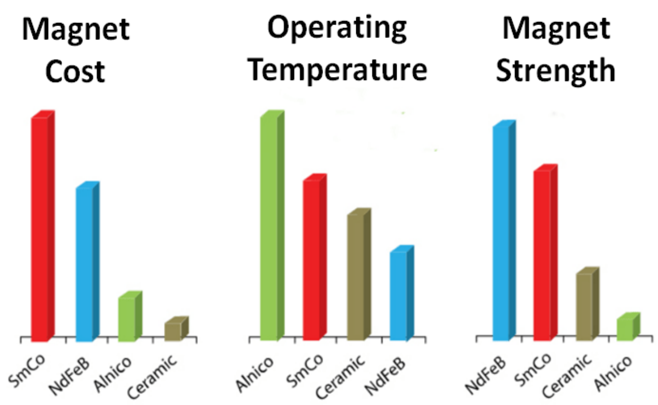


https://www.electronics-tutorials.ws/electromagnetism/magnetic-hysteresis.html

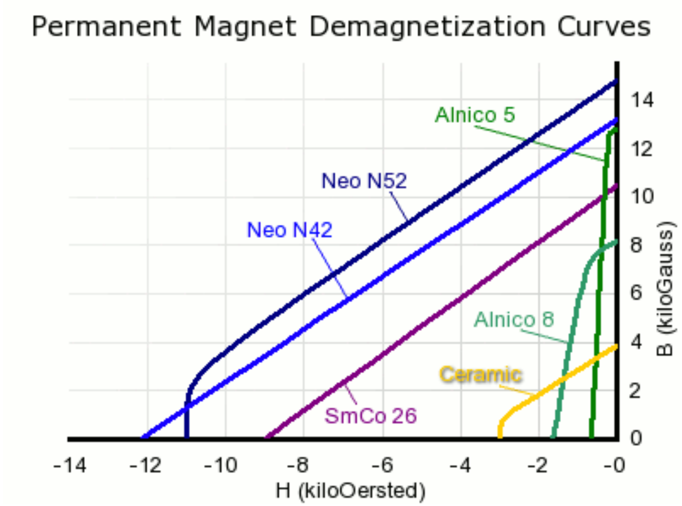
- Ferromagnets: there is a saturation of B (of order 1 T)
- Ferromagnets have very high μ (make a good core) $\mu/\mu 0 > 5000$
- "hard" ferromagnet: high Coercive field= keep large **m** when external field is turned off.
- "Soft" has low Coercive field, can change magnetic polarity even at low applied field

https://ocw.mit.edu/courses/materials-science-and-engineering/3-024-electronic-optical-and-magnetic-properties-of-materials-spring-2013/lecture-notes/MIT3_024S13_2012lec25.pdf

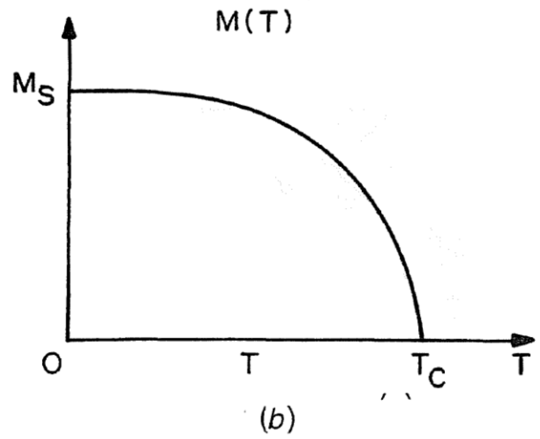
Various grades of “hard” ferromagnets



https://www.duramag.com/techtalk/tech-briefs/magnet-materials-comparison-guide/



https://www.kjmagnetics.com/images/blog/demag.curves2d.gif



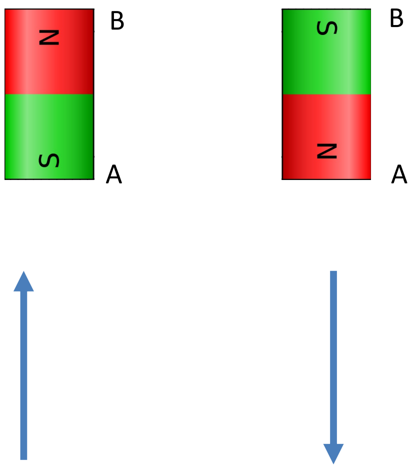
O’Handley, Robert C. Modern Magnetic Materials: Principles and Applications. New York: Wiley, 2000.

- Ferromagnets differ in maximum remanence, coercivity, and Curie temperature depending on the material used
- Manufacturers datasheets often provide the second quadrant of the magnet’s BH curve
- For ex. NdFeB are strong but have low curie temperatures and are thus not suited for applications above 80°C (such as happens in some EM actuators…)

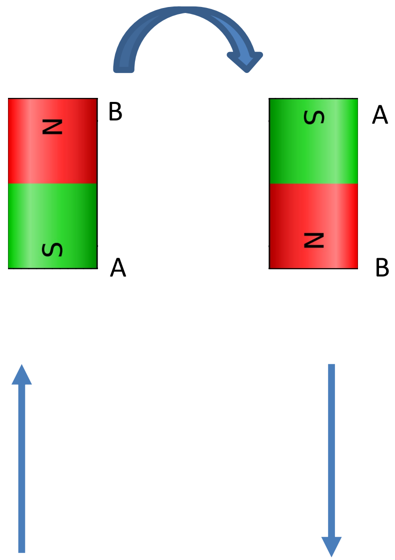
In a <u>uniform</u> field, only a torque is generated on a magnet

- What happens to “soft” ferromagnet in a uniform (rather low) B field?
    - internal magnetisation changes to match the applied field
- What happens to “hard” ferromagnet in a uniform (rather low) B field?
    - Rotates to lines up with external field

When the applied field flips, the soft ferromagnet does not physically rotate, but takes on the opposite magnetisation.



Rapidly flip the external magnet orientation



hard ferromagnet rotates in space to follow the applied field.

Rapidly flip the external magnet orientation

$$\tau_m = m \times \vec{B}$$

«magnetic insect» : exploits both soft and hard Magnetic materials

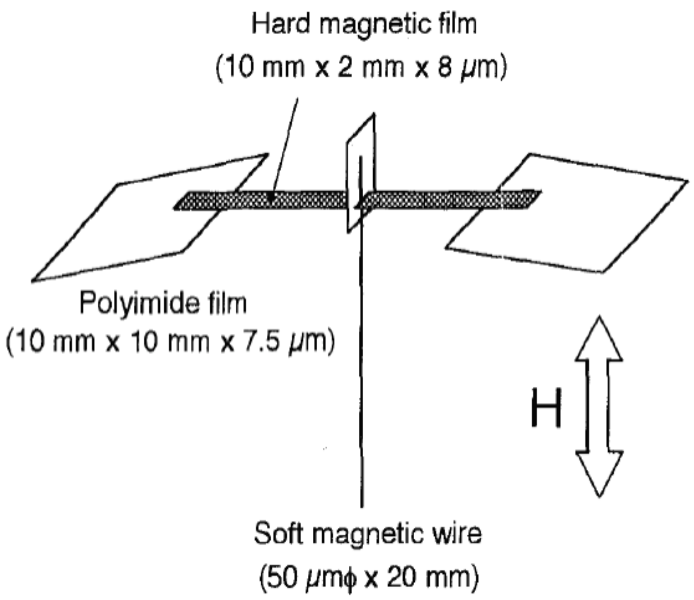


Figure 1. Schematic view of the flying machine.

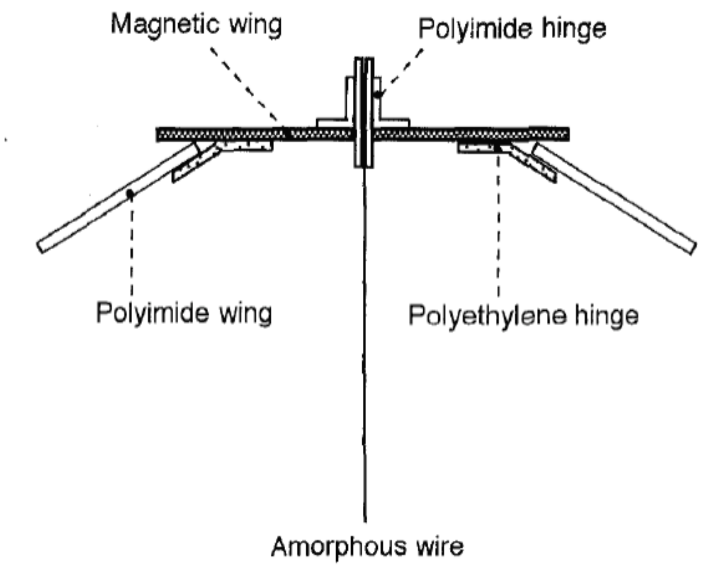
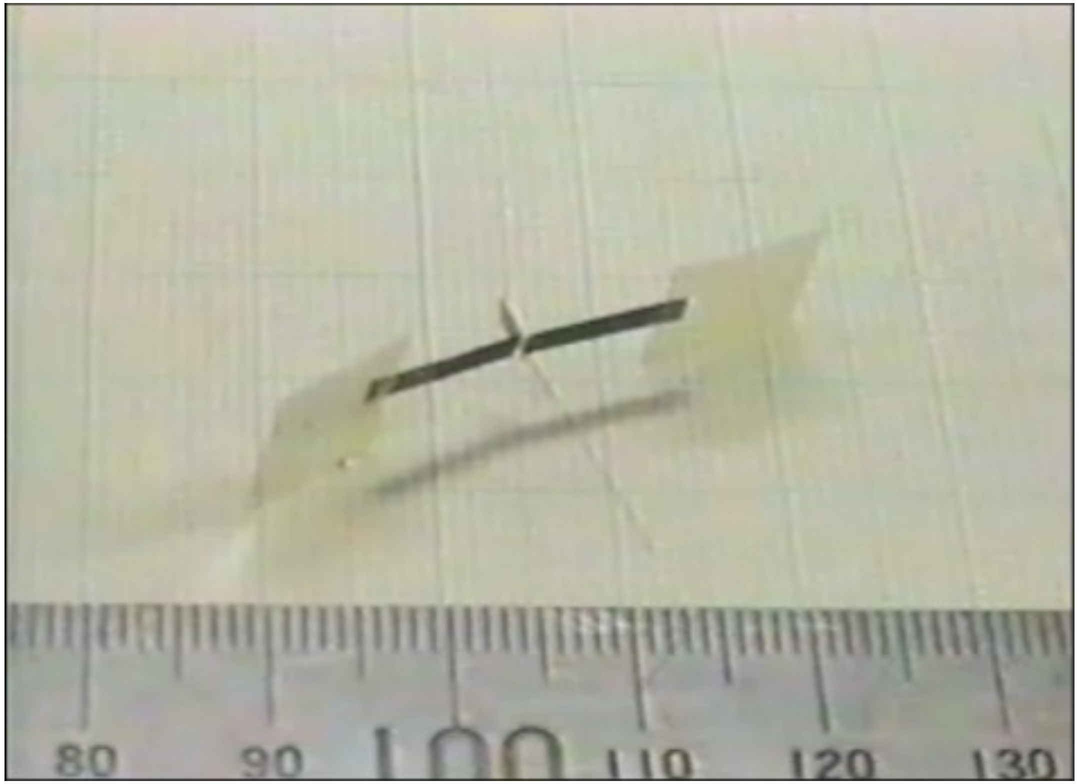


Figure 2. Side view of the flying machine.

K. I. Arai, et. al: "Fabrication of small flying machines using magnetic thin films", IEEE Trans. Magn., Vol. 31, No. 5, pp.3758-3760 (1995).

http://www.youtube.com/watch?v=PbUoYHgv8-Y

«The magnetic insect»



K. I. Arai, et. al: "Fabrication of small flying machines using magnetic thin films", IEEE Trans. Magn., Vol. 31, No. 5, pp.3758-3760 (1995).

http://www.youtube.com/watch?v=PbUoYHgv8-Y

2 types of Source of magnetic field

Permanent magnet:

- high field (but max approx. 1 T)
- zero energy consumption
- high field gradient close to the magnet edges or if the magnet is small
- but
  - cannot modulate the field strength (unless move the magnet)
  - design can be challenging for arrays

Coil (or wire):

- For small coils, cannot reach 1 T (unlike larger coils, eg for MRI)
- electrical control of magnetic field: can quickly change field strength and possibly orientation
- Field shape set by coil or wire geometry
- but
  - field strength limited by max. current,
  - high power consumption…

- Large coils enable action at a distance, eg for remotely moving object in body

Maximum Current in a Wire: 5 options

W. Trimmer, ‘‘Microrobots and micromechanical systems,’’ *Sensors and Actuators*, 19, p.267 (1989)

J. Peirs *et al,* "Scale effects and thermal considerations for micro-actuators", *Robotics and Automation,* 1998.

1. Assume that **current density (I/A)** is an intensive variable (indep. of size), then $\boldsymbol{I \sim L^2}$

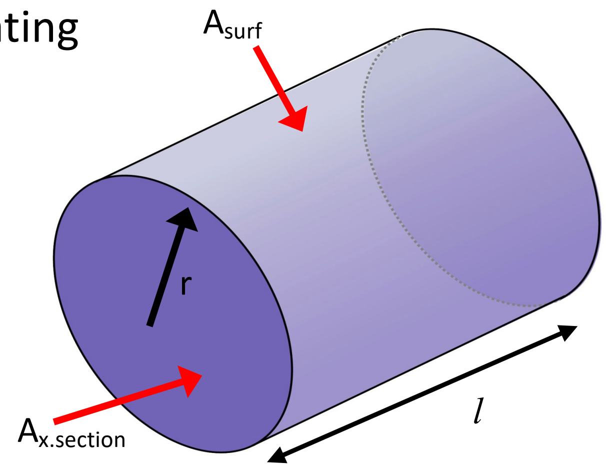
2. Assume Heat flow out of the wire through surface is equal to Joule heating

   (i.e. $\dot{Q}/A_{surf} = const$)

$$\dot{Q} = P = I^2R = \mathrm{I}^2\left(\frac{\rho l}{A_{x-section}}\right)$$

$$\frac{\dot{Q}}{A_{surf}} = \frac{I^2\rho l}{A_{x-section}A_{surf}} \propto \frac{I^2 l}{r^2 l.r} = I^2L^{-3}$$

So then $\boldsymbol{I \sim L^{1.5}}$



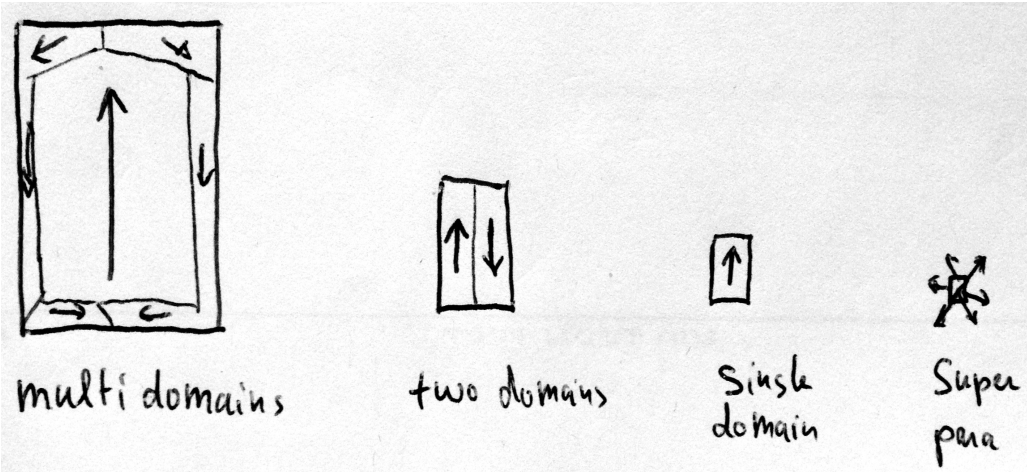
3. Assume constant temperature rise ΔT of the wire, independent of size.
If wire resistivity ρ and thermal conductivity k_{th} are intensive, the balance between conductive heat dissipation and Joule heating $\dot{Q} = k_{th}\frac{\Delta T}{\Delta r}A_{surf} = \rho\frac{l}{A}I^2$ implies that $\boldsymbol{I \sim L}$

(4. Electromigration when I >10^{10} A/m^2)

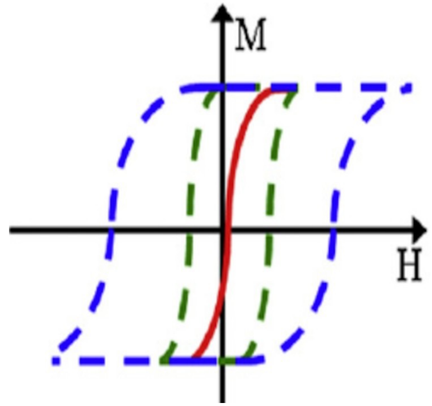
(5. if Convection *h* is important compared to conduction, then $I \propto L^{1.5}$)

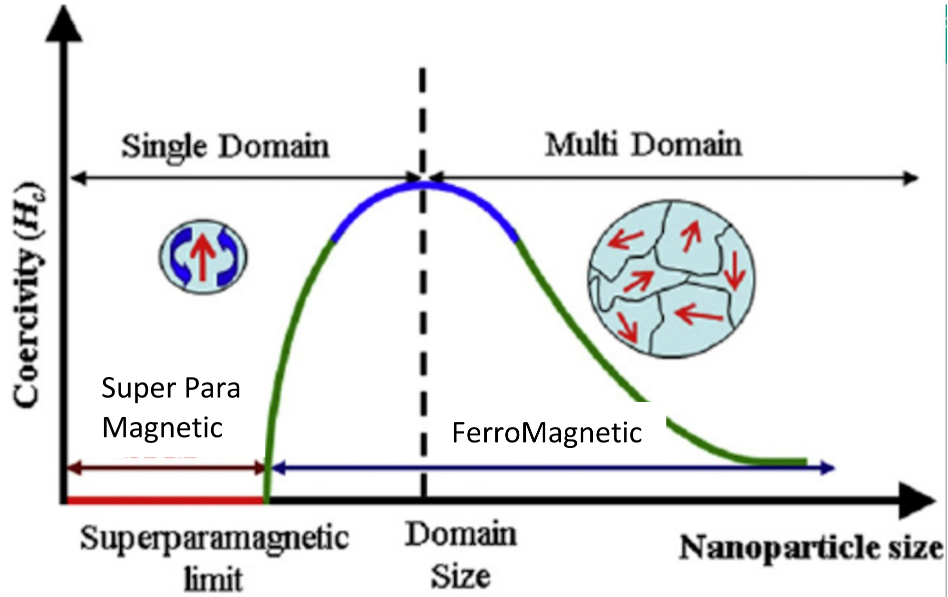
EM2: Scaling of magnets

EM2: Scaling of magnets



When the magnet becomes small enough, it turns into a **single domain magnet**.





The transition to a single domain occurs when:
the energy of "wall boundary + anisotropy energy"
becomes higher the "single domain" energy.

Scaling of small single domain magnets : the superparamagnetic transition

Energy of a single domain particle: $$E_{mag,1domain} = \frac{4\pi}{3} r^3 \mu_0 M_s^2$$

M_s : saturation magnetization

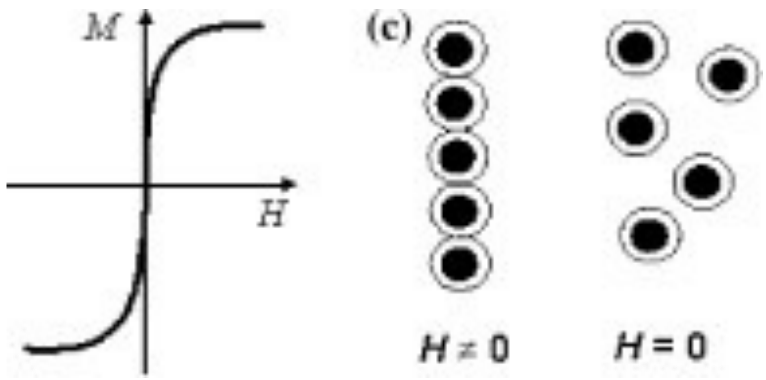
A particle become **superparamagnetic** when its **magnetic energy** is comparable to **thermal energy:**

$$E_{mag} << k_B T \qquad k_B T = 4 \cdot 10^{-21} J \quad at\ 20°C$$

For Fe particles, the superparamagnetic transition occurs around $d_{SP} \cong 10\ nm$

In the absence of external magnetic field, unlike macro magnetic particles, **superparamagnetic particles do not attract each other** (as they have no net magnetisation)

In presence of external magnetic field, **superparamagnetic particles are magnetized and form chains.**

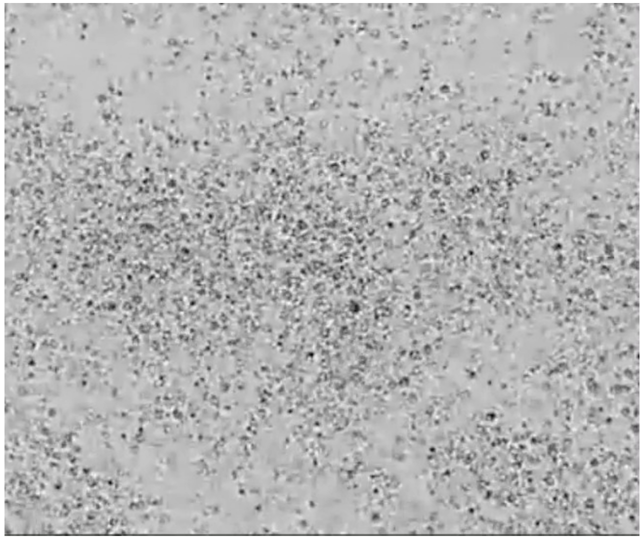


M. Gijs, Microfluid. Nanofluid 1 (2004) 22

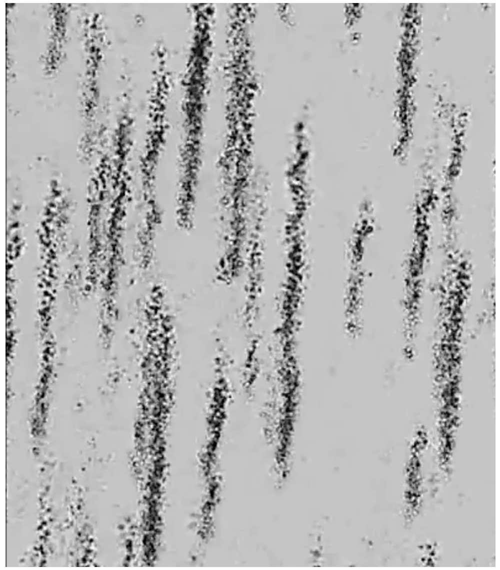
! Superparamagnetic Particle are **not** rotating in space, only M is rotating due to thermal energy

Paramagnetic particles

Turn on the B field



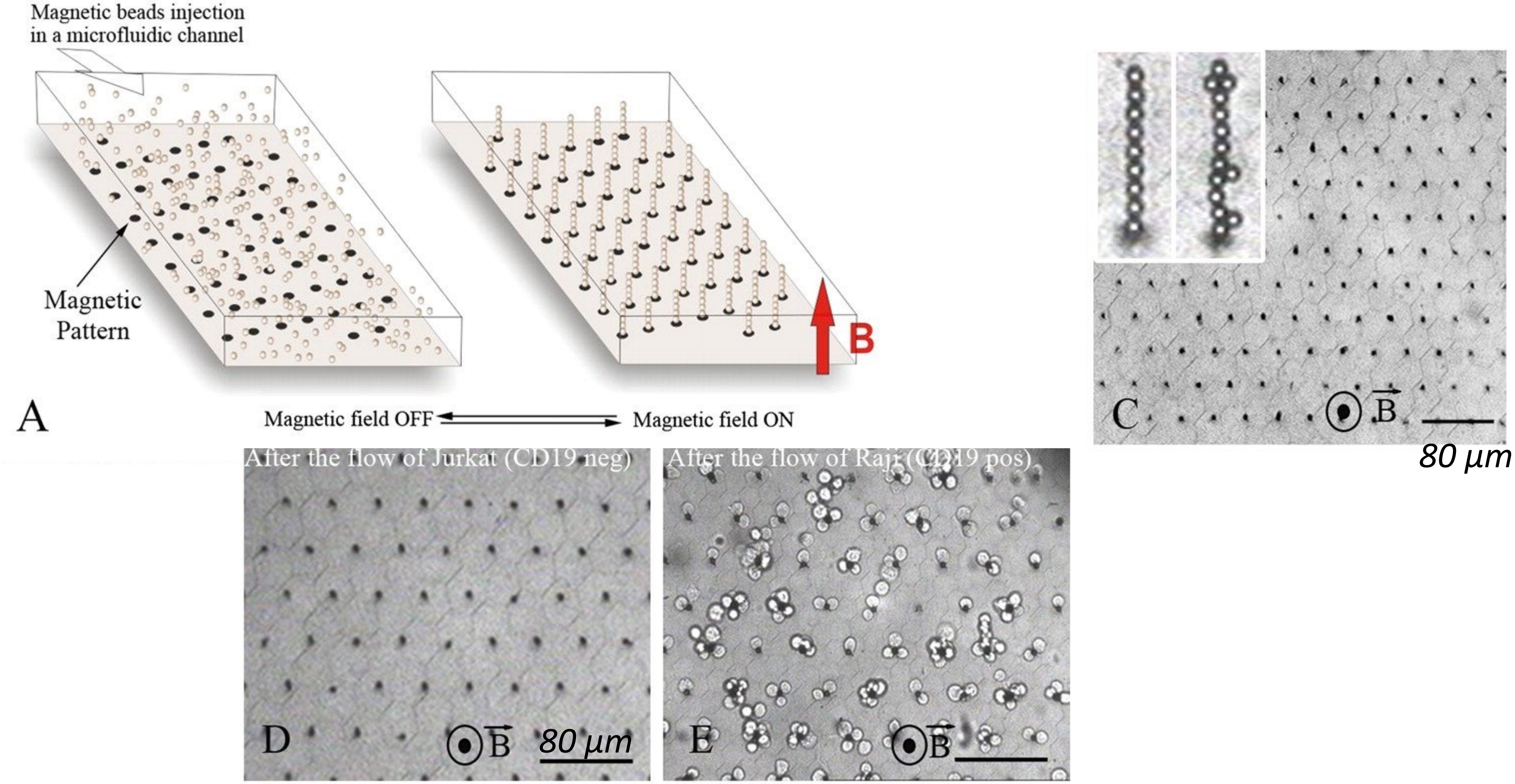
Turn off the B field



Diffusion…

Magnetic bead self-assembled columns

Superparamagnetic beads coated with an antibody are injected in the channel



Saliba, A.-E. *et al.* Microfluidic sorting and multimodal typing of cancer cells in self-assembled magnetic arrays. *Proceedings of the National Academy of Sciences* **107**, 14524–14529 (2010).

EM3: Magnetic Storage (scaling)

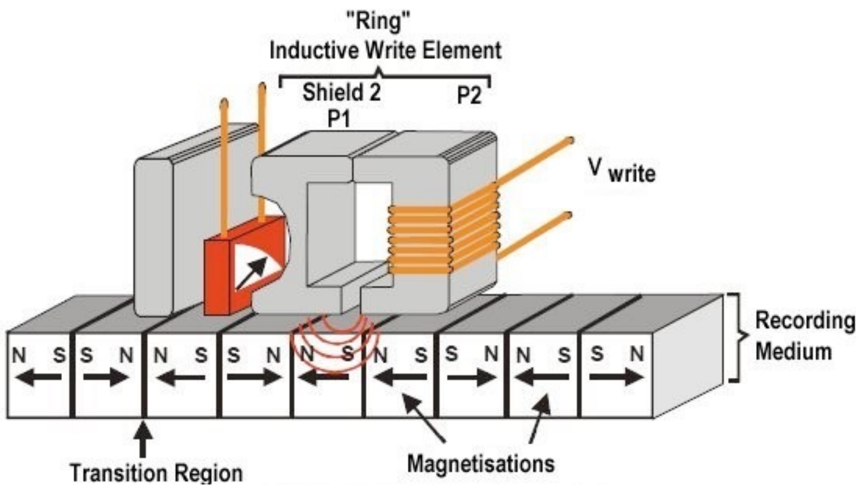
Magnetic storage – longitudinal recording (the "old" way)

In magnetic recording, the anisotropy energy $V.K_U$ helps to maintain the magnetization in one specific direction.

K_U is linked to coercivity $H_c=2K_u/M_s$

V: volume of the bit; K_U: anisotropy constant
M_s : saturation magnetization; H_c: switching field

$$\text{typical } K_U \approx 10^4 \, J/m^3$$



Smaller magnetic domains have smaller anisotropy energy…

When the thermal energy, k_BT, approaches $V{\cdot}K_U$, the magnetization becomes unstable.

The stability time is given by $\tau = \tau_0 e^{\frac{K_U V}{k_B T}}$ where τ_0 is the precession time (about 1ns)

$$\ln\left(\frac{\tau}{\tau_0}\right) = \frac{K_U V}{k_B T}$$ for t= 10 years, we find that $\frac{K_U V}{k_B T} \geq 60$

So, want higher K_U to allow smaller V if we aim for $K_U V > 60\ k_B T$

Storage Density limited for longitudinal recording to: 0.1 Tb/in^2

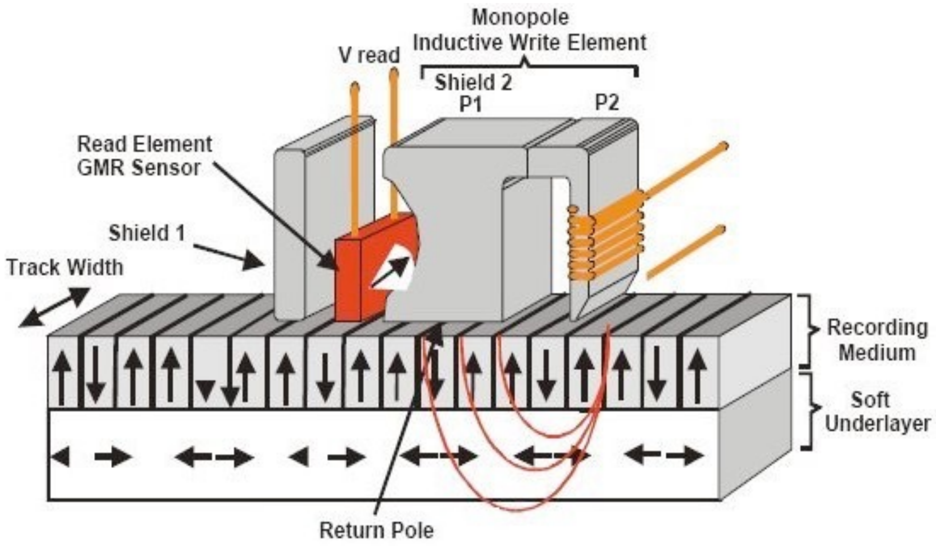
Magnetic storage – perpendicular recording (the “newer” way)

In vertical recording, special media with grain and multilayer have an out-of-plane magnetic anisotropy

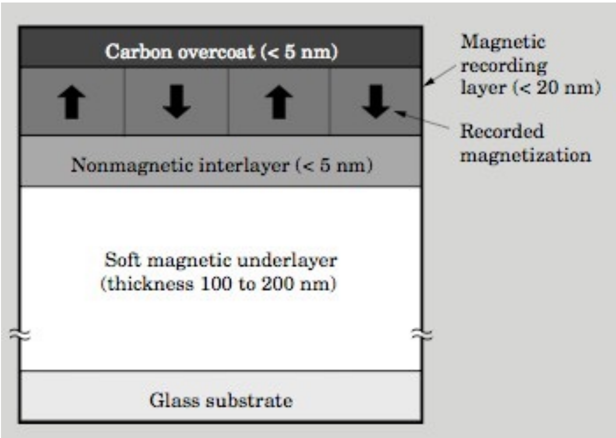
The soft magnetic underlayer helps to maintain magnetization and increase gradient when writing (allows for higher K_U material).

Higher K_U means smaller domain while still not superparamagnetic.

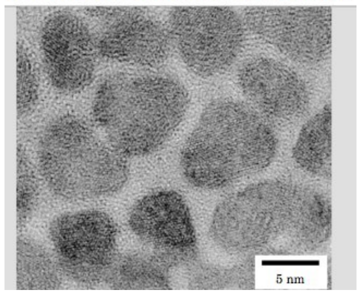
(in practice, this was made possible by GMR readhead)



CoCrPt alloy films have high magnetic anisotropy



TEM image of CoCrPt-SiO_2 granular magnetic layer

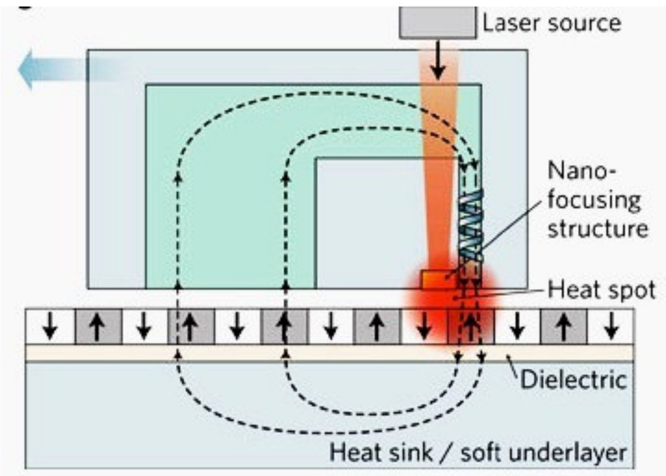


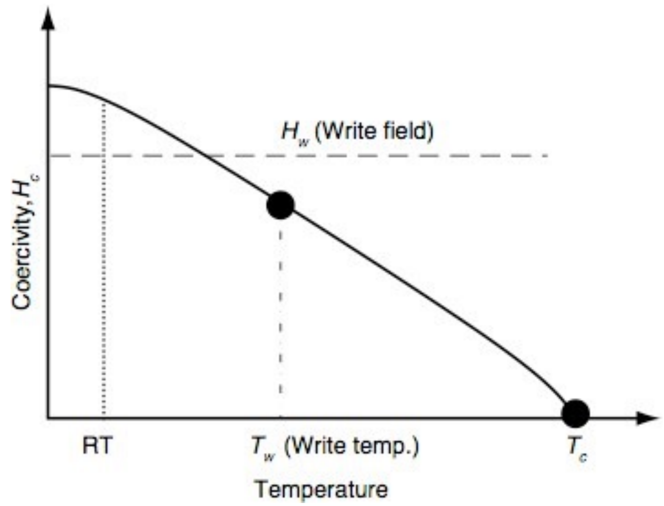
Can reach Storage Density: 1 Tb/in^2

Magnetic storage – heat assisted magnetic recording – HAMR (Seagate 2020)

To reach Higher K_U implies that a higher writing field: the switching field H_c of a material is proportional to K_U and inversely proportional to the saturation magnetization M_s of the recording medium (for a single particle, $H_c=2K_u/M_s$)

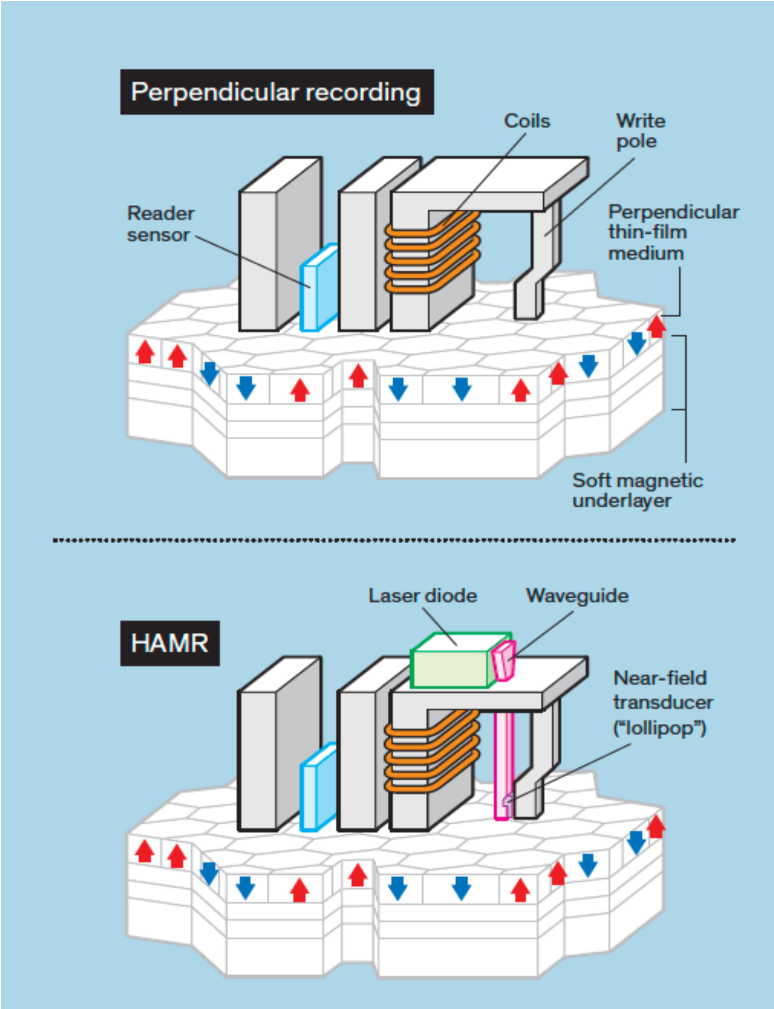
The writing field that can be produced by a head is limited by the saturation magnetization of the material used in the head.



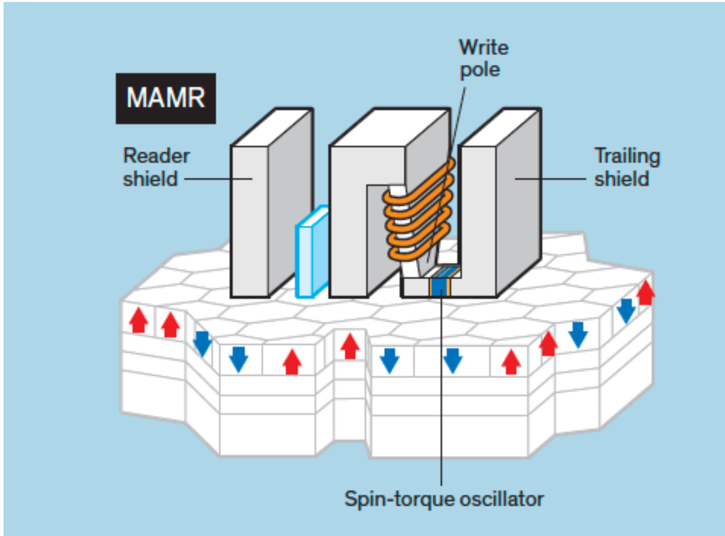


Local heating (here by focused laser light) allows locally and temporarily decreasing H_c
Near field optics confine the spot to sub-50 nm diameter

Storage Density: 10 Tb/in^2



Or heat using microwaves:
MAMR (micro-wave assisted magnetic recording)
since 2019 from Western Digital



A. Nordrum, The fight for the future of the disk drive. *IEEE Spectrum*. **56**, 44–47 (2019).

EM4: Magnetic forces

1. Scaling of magnetic forces on *small magnetic beads*

2. Scaling of magnetic forces of *wires on wires* or *wires on magnets*

4.1 Magnetic forces on beads

- in **homogeneous magnetic field,** a ferromagnetic bead (not superparamagnetic!) feels a **torque,** but no force
- in **magnetic field gradients,** a ferromagnetic and a superparamagnetic bead sense a net **force**
- Magnetic force scales with the **volume** of the particle

$$m = V \cdot \chi_{bead} \cdot H$$

$$\boxed{F = (\vec{m}.\nabla)\vec{B}}$$

$$\vec{F}_{mag} = \frac{1}{\mu_0} V \chi_{bead} \left(\vec{B} \cdot \nabla\right) \vec{B} \quad + \underbrace{\rho V \nabla\left(\vec{M}_0 \cdot \vec{B}\right)}_{<<}$$

χ: *magnetic susceptibility* *(about 1- 3)*
B: *magnetic induction ($B=\mu_0 H$)*
V: *volume of the bead*

The left term is due to the magnetization aquired through the high **magnetic susceptibility**.
The right term is due to initial magnetization M_0 of the bead, it is ***very small***

F scales with: **(bead volume)** x **(field magnitude)** x **(field gradient)**

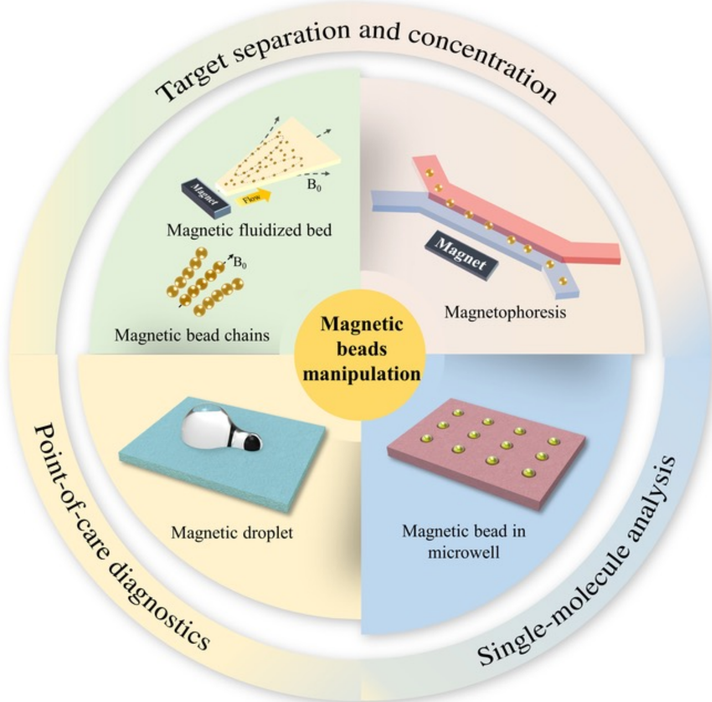
Why do we care about magnetic beads?

- Can *coat* them (metal, silica, carbon, polymer)
- Can *functionalize* them (catalysis, DNA-specific, antibody, peptides, etc)
- Can *Label* beads

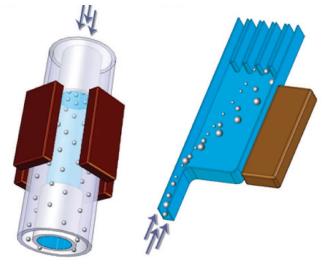
- High surface to volume ratio

- Can magnetically *remotely* manipulate the beads, e.g., to sort for analysis, or to stretch DNA

- Can make chains

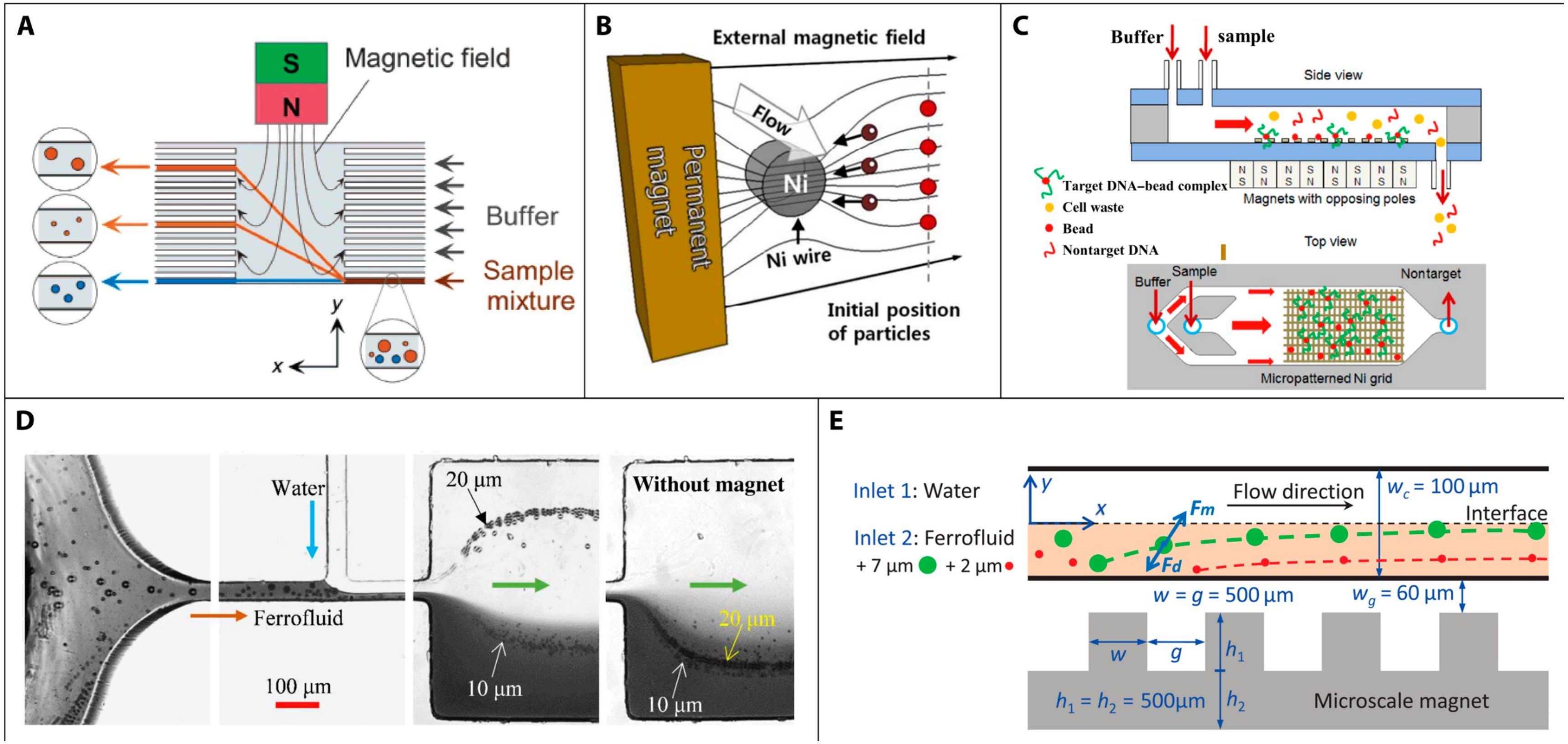


G. Cai et al, Magnetic Bead Manipulation in Microfluidic Chips for Biological Application. *Cyborg and Bionic Systems* **4**, 0023 (2023).



M. A. M. Gijs, et al, "Microfluidic Applications of Magnetic Particles for Biological Analysis and Catalysis", *Chem. Rev.* **110**, 1518–1563 (2010).

Magnetophoresis

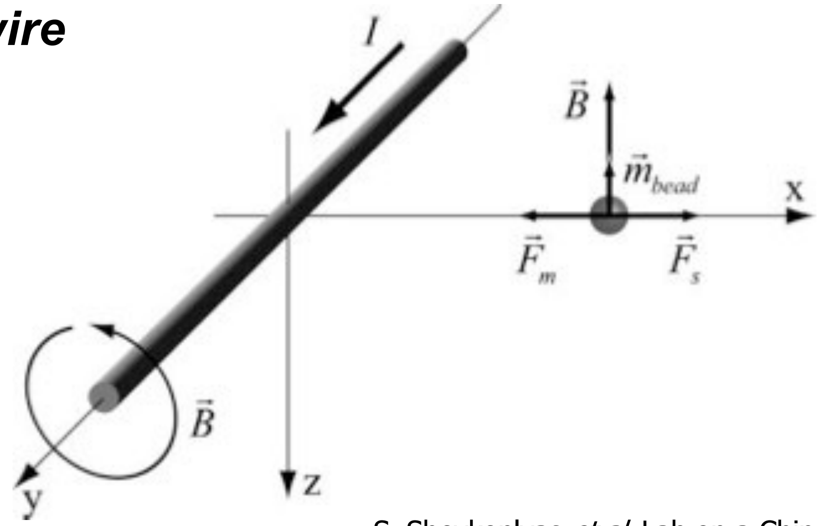


(A) Using free-flow magnetophoresis for separation magnetic beads with different sizes.
(B) Magnetophoresis using a ferromagnetic nickel wire.
(C) The target DNA–bead complexes are absorbed on the edge of the nickel grid, while the nontarget DNA flows directly out of the exit
(D) Continuous separation of 20-µm particles from 10-µm ones in a T-shaped microchannel.
(E) Principle of the focusing and separating diamagnetic beads using 2 coflowing fluids.

G. Cai et al, Magnetic Bead Manipulation in Microfluidic Chips for Biological Application. *Cyborg and Bionic Systems* **4**, 0023 (2023).

Magnetic forces on a bead due to current flowing in a wire

Pulls the bead to the wire



S. Shevkoplyas *et al*, Lab on a Chip, 2007, 7, 1294–1302

For a field gradient in the *x* direction (neglecting bead initial magnetization M_0):

$$F_{mag,x} = \frac{1}{\mu_0} \cdot V \cdot \chi_{bead} \cdot B \frac{\partial B}{\partial x}$$

Both the B Field and B field gradient are created by the wire at distance x:

$$B_x = \frac{\mu_0 I}{2\pi x} \qquad \frac{dB_x}{dx} = -\frac{\mu_0 I}{2\pi x^2}$$

$$\boxed{F_{mag,x} = V \cdot \chi_{bead} \cdot \frac{\mu_0}{4\pi^2} \frac{I^2}{x^3}}$$

High forces for very short distances from the wire

Assuming scaling with constant current density in the wire ($I \propto L^2$), the scaling of magnetic force with system size *L* (assuming same particle size) is:

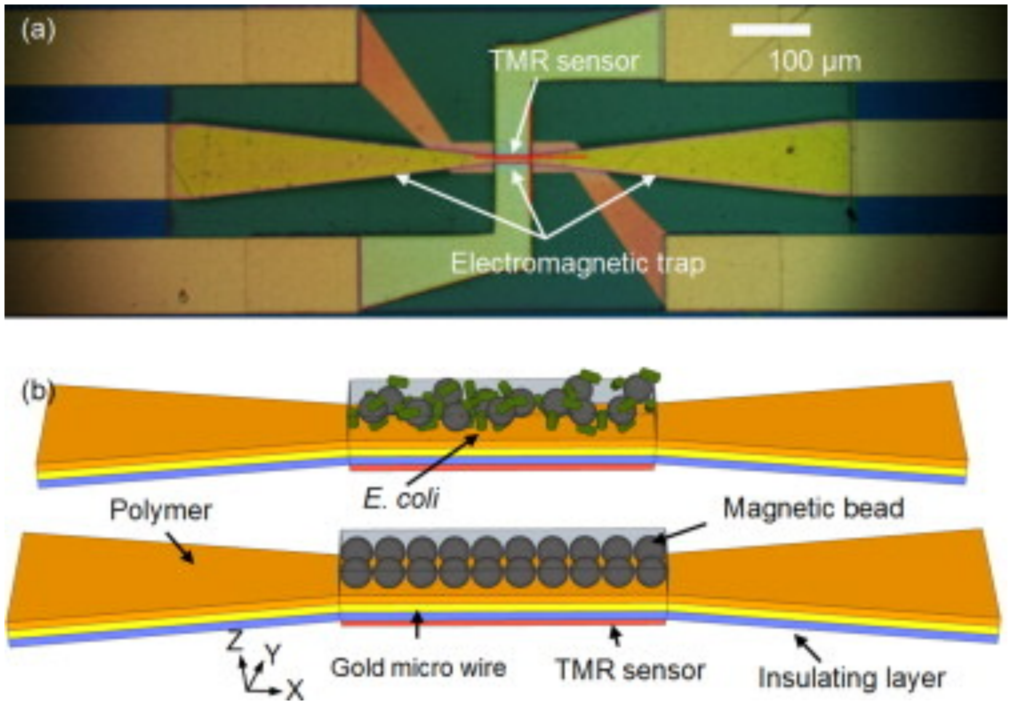
$$\boxed{F_{mag} \propto V_{bead} \frac{L^4}{L^3} \propto V_{bead} L}$$

smaller bead : smaller force…
And
smaller system : smaller force…

Example of ElectroMagnetic actuation on beads from a wire

"experiments were carried out with a 6 µm wide microwire, which attracted the magnetic beads from a distance of 60 µm, when a current of 30 mA was applied."

No permanent magnet



[An efficient biosensor made of an electromagnetic trap and a magneto-resistive sensor (2014)](https://doi.org/10.1016/j.bios.2014.03.035)
https://doi.org/10.1016/j.bios.2014.03.035

For higher forces, we want Simultaneously:

- High: **B**

and

- High: $\boldsymbol{\nabla B}$

Till now, one wire generated both B and $\boldsymbol{\nabla B}$

Solution

- Add a permanent magnet to generate **high B**

and

- Use small coils / wires to generate (and tune) a **high** $\boldsymbol{\nabla B}$

Manipulation of beads with planar coils and permanent magnets (M. Gijs' lab) :

Since the magnetic field magnitude generated by a coil is limited at small scale, we **superimpose a larger static uniform magnetic field** (B_0 from an external permanent magnets) on the small varying field produced by the coils.

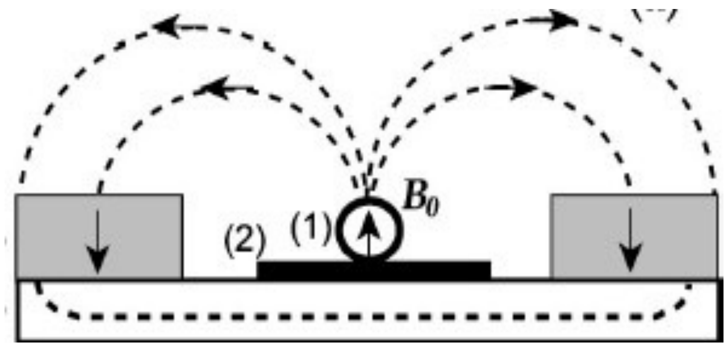
Force without B_0 : $F_1 \propto B_{coil} \frac{\partial B_{coil}}{\partial x}$

Force with B_0 : $F_2 \propto \left(B_{magnet} + B_{coil}\right) \frac{\partial B_{coil}}{\partial x}$

The force gain is : $F_2 / F_1 \approx B_0 / B_{coil}$

The scaling becomes: $\boxed{F_{mag} \propto V_{bead} \frac{L^2}{L^3} \propto V_{bead} L^{-1}}$

Much more favorable downscaling when combine permanent magnet with small coil !



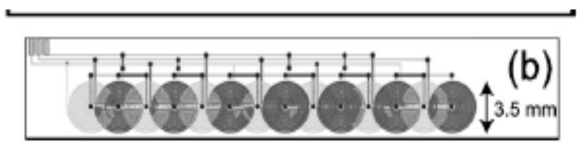
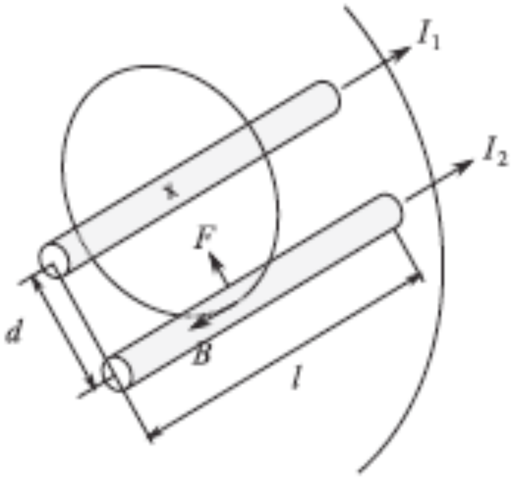


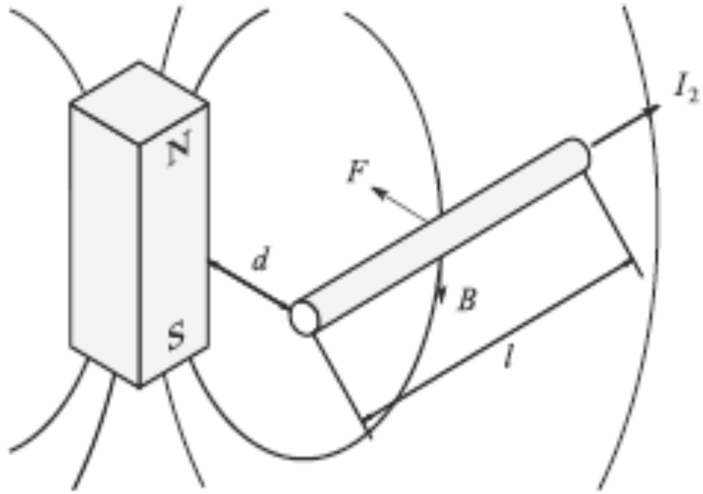
FIG. 1. (a) Schematic view of the magnetic microbeads transport device. Two permanent magnets (3), symmetrically disposed on a metallic sheet (4), generate a large and uniform magnetic field B_0 at the position of the microfluidic capillary (1), thereby strongly enhancing the magnetization of the magnetic beads. The small time-dependent magnetic field of an array (2) of simple planar coils allows then transport of the beads in the capillary. (b) Layout of the array of planar coils (2) with partially overlapping coils arranged over two PCB layers.

Rida et al., Appl. Phys. Lett 83, 12 (2003)

4.2 ElectroMagnetic actuation (not beads)



current – current (or wire-wire)



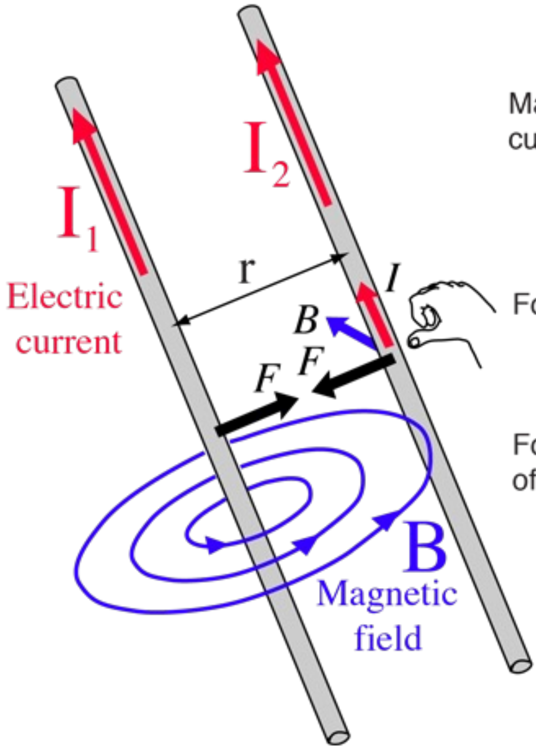
current – magnet

Table 1 Force scaling law of magnetic actuators.

	current-current	current-magnet
$I \propto L^2$	L^4	L^3
$I \propto L^{1.5}$	L^3	$L^{2.5}$
$I \propto L^1$	L^2	L^2

Liu, D. K.-C., Friend, J. & Yeo, L. "A brief review of actuation at the micro-scale using electrostatics, electromagnetics and piezoelectric ultrasonics". *Acoust. Sci. & Tech.* **31**, 115–123 (2010).

Electromagnetic actuation scheme: wire-wire



Magnetic field at wire 2 from current in wire 1:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

Force on a length ΔL of wire 2:

$$F = I_2 \Delta L B$$

Force per unit length in terms of the currents:

$$\frac{F}{\Delta L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

$$\boxed{F_m = \frac{\mu_0}{2\pi} I_1 I_2 \frac{L}{r}}$$

Assuming constant current (ΔT):

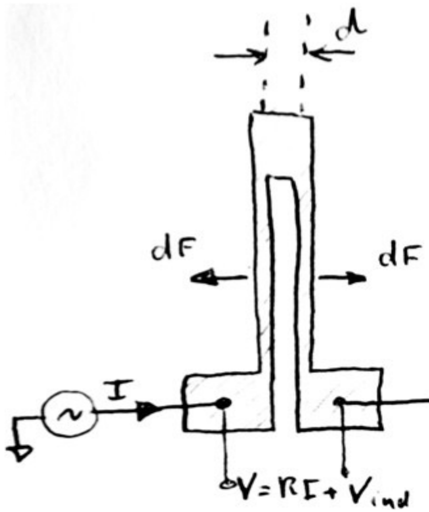
$I \propto L \qquad F_m \propto L^2$

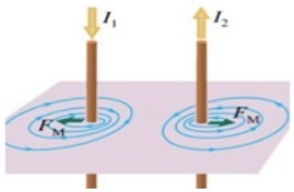
Assuming Constant current density:

$I \propto L^2 \qquad \boldsymbol{F_m \propto L^4}$

http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/wirfor.html

Double beam resonator: scaling of sensing *(=wire-wire scheme)*





Magnitude of field produced by a current at distance *d*:

$$B = \frac{\mu_0 I}{2\pi d}$$

Force exerted on the second beam:

$$|dF| = \frac{\mu_0 I^2}{2\pi d} \cdot dy$$

$$x_{\max} = \frac{\mu_0}{2\pi\, d} \frac{I^2 \cdot l}{\omega_0 \cdot \lambda}$$ and $$|V_{ind}| = \left(\frac{\mu_0}{2\pi d}\right)^2 \frac{I^3 \cdot l^2}{2\lambda}$$

Usual Problem of Joule heating:

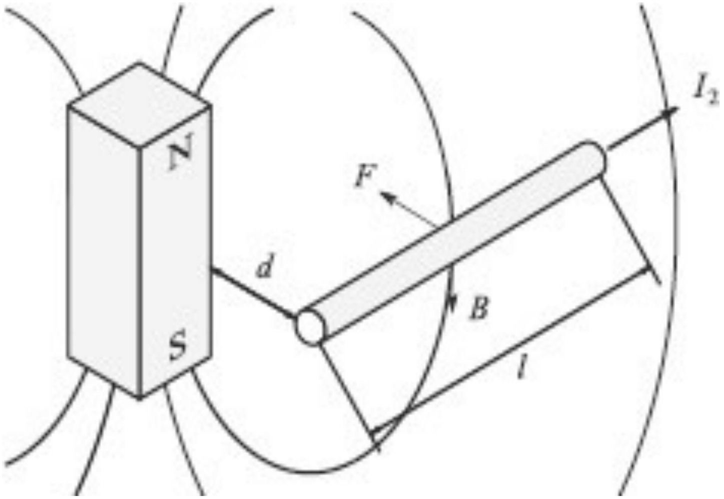
We cannot keep the same current when downscaling because of Joule heating. Can assume $I \propto L^2$

$$|V_{ind}| \propto \frac{L^6 \cdot L^2}{L^2} \propto L^6$$

this is a very unfavorable downscaling law

(in later slides, where the B field is produced by a magnet, we will see that scaling is a bit better $\boxed{V_{ind,magnet} \propto L^4}$

Electromagnetic actuation schemes: Wire-magnet



$$dF = I\, dl \times B,$$

$$F_m = B I_2 l$$

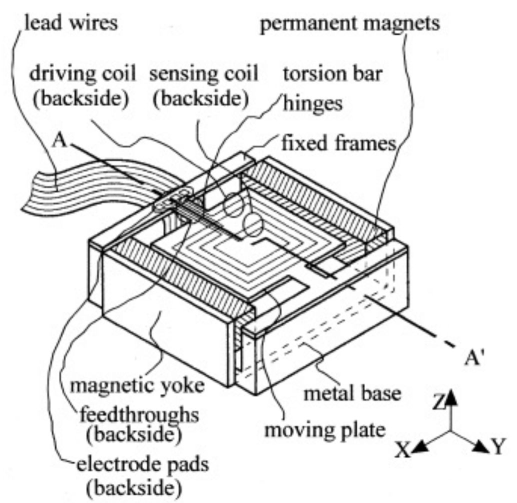
(highly simplified, assumes uniform B)

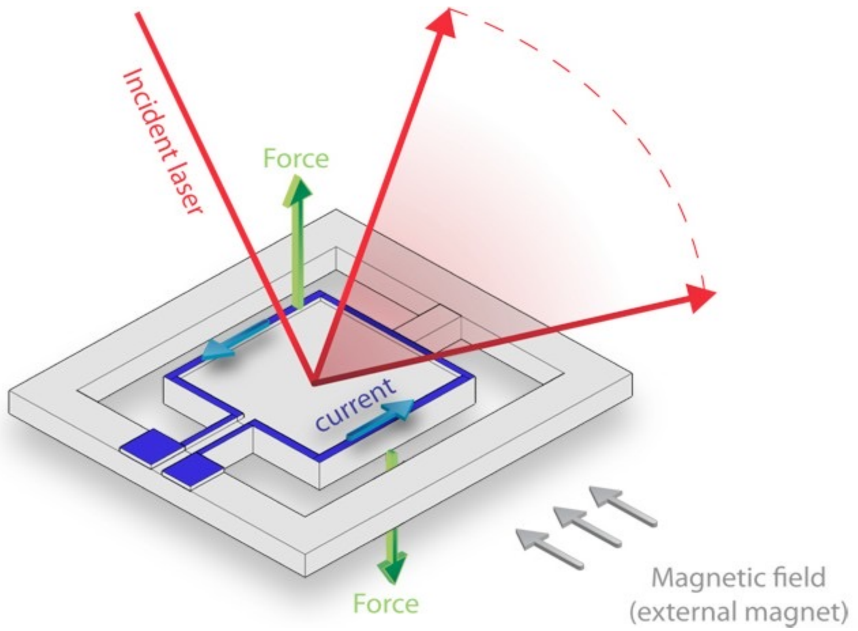
Assuming constant current $\quad I \propto L^0 \quad F_m \propto L^1$

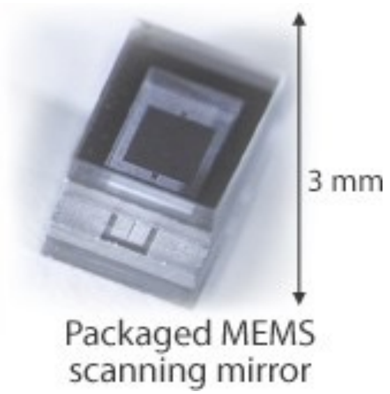
Assuming Constant current density: $\quad I \propto L^2 \quad F_m \propto L^3$

Electromagnetic micro-mirror *(=wire-magnet scheme)*

The Lorentz force produced by current in a wire. A constant magnetic field (co-planar with the mirror) comes from a set of permanent magnets (carefully arranged)

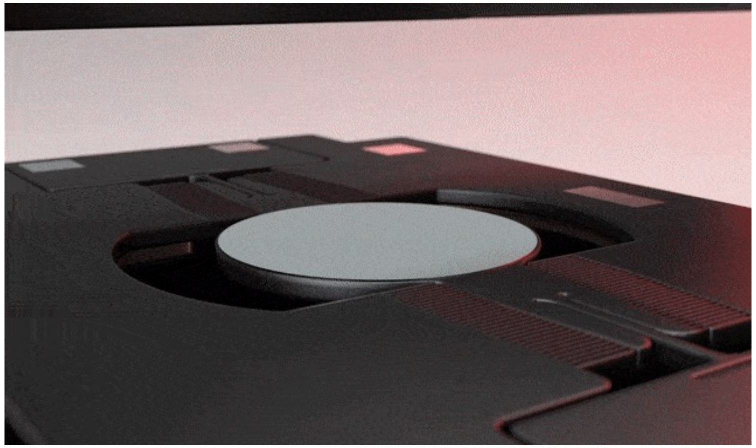






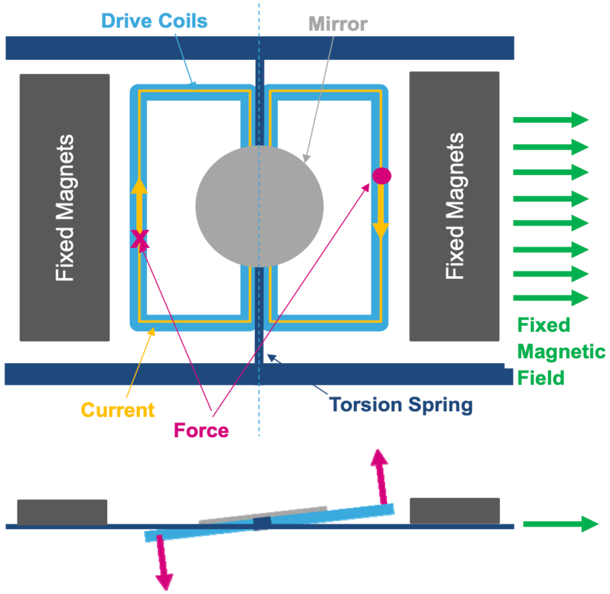
- First report: "A MEMS electromagnetic optical scanner for a commercial confocal laser scanning microscope", Miyajima e al., J. Microelectromechanical Systems, 12, (2003) p243. DOI: 10.1109/JMEMS.2003.80996
- Lemoptix (ex. EPFL 2008), purchased by Intel (2015), then by Magic Leap (2018), then ….
  F_{res}: 3 to 50 kHz, angles up to 30° at resonance for a few V

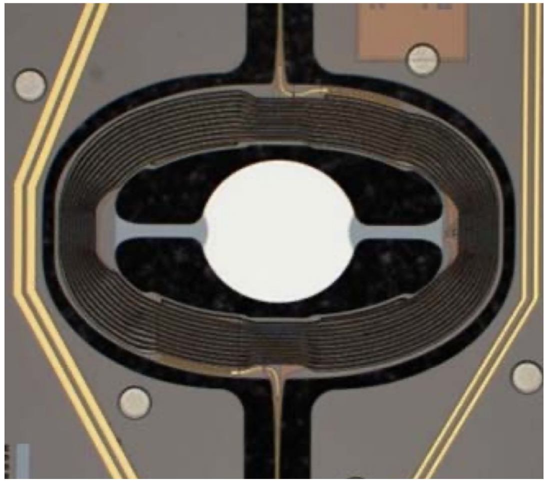
Electromagnetic micro-mirror developed by several companies

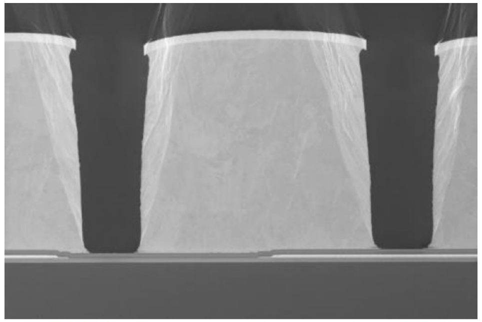


https://www.st.com/content/st_com/en/about/innovation---technology/laser-beam-scanning.html

ST microelectronics

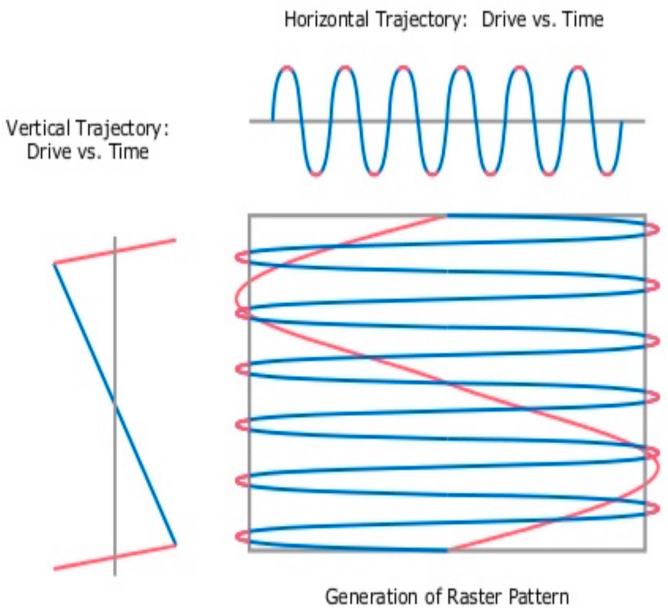


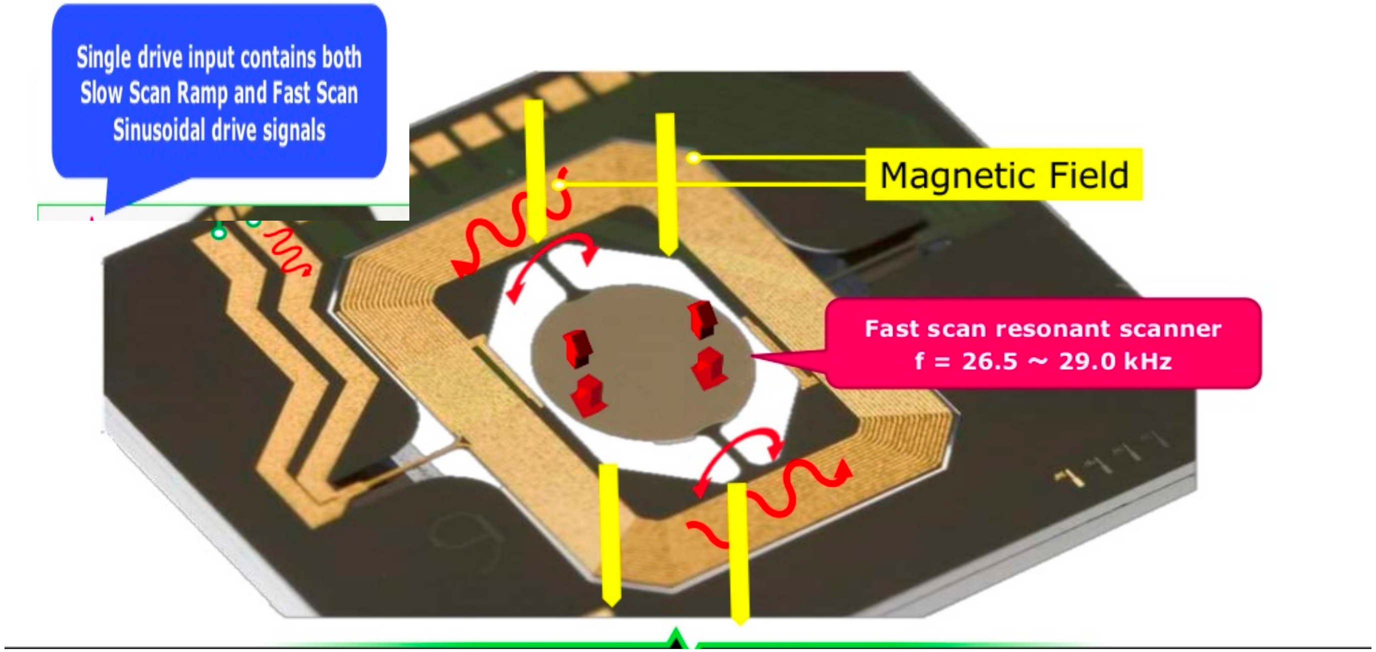




Coil Wire for magnetic actuation. Thick ECD growth (>20um)

2 scan axes with only one coil: exploit different resonance frequencies





Microvison (was initially for pico-projectors, now developing LIDAR)

Davis, W.O., Sprague, R., Miller, J., "MEMS-based pico projector display", Optical MEMs and Nanophotonics, 2008 IEEE/2008, pp. 31 - 32

Complex trade-offs…

Want:

- high frequency
- Flat mirror
- Self-sensing of angle
- Two axis
- Low power & low heating

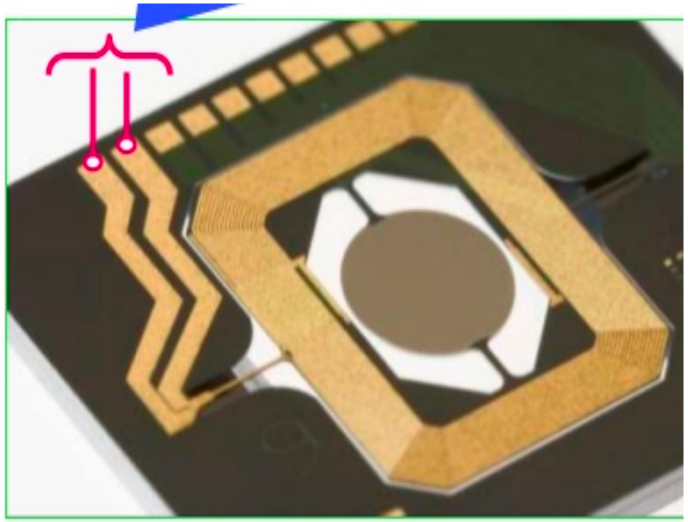
Stiff spring

-Thick mirror (heavy, so need even stiffer spring…)

- No coils in the optics

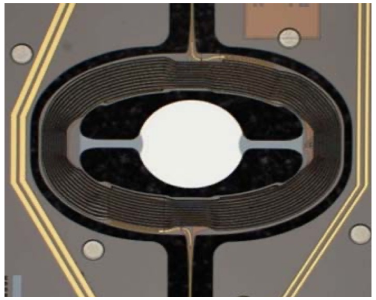
Read the induced voltage for feedback

Dual gimbal, two coils? Or One coil?

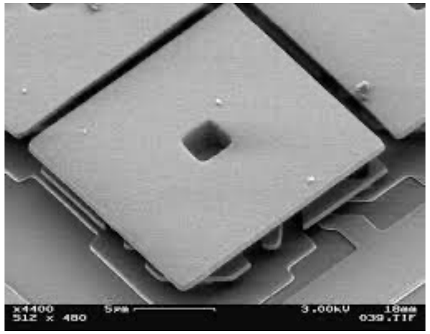


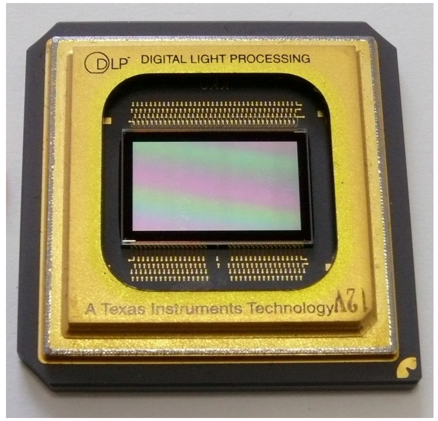
- Low resistance coil (so thicker metal, heavier)
- Many turn to get high Lorentz force (but then heavier)
- High-field permanent magnet

EM scanning mirror vs. DMD array



$$F_m = B I_2 l$$





EM:

works better at larger scale: a mm to cm size mirror,
at 50 Hz to 10s of kHz

Gap to magnet approx 1 mm
Drive at 1-2 V

$F = B\ I\ L_{wire}$ = 5.10^{-3} N = 5 mN

Assume 2 cm diameter coil, 20 turns, 0.1 T field, and I=40 mA

Control: raster on 2 axis

How small can one go?

ES:

works better at smaller scale: move a µm-size mirror, at >100 kHz

Gaps of few µm
Drive at 20 V

$$F_{\parallel plate} = \frac{\frac{1}{2}\epsilon_0 \mathrm{V}^2 A}{d^2}$$

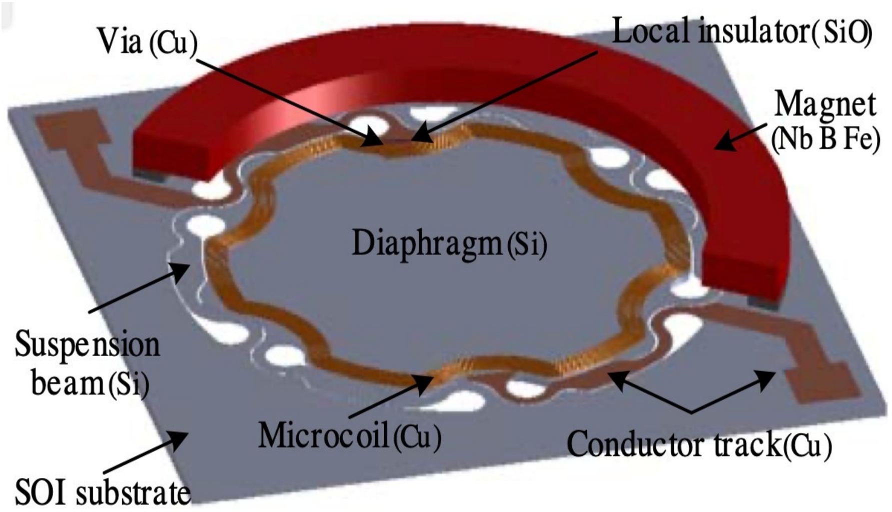
= 2.10^{-8} N per pixel or
= 20 mN per megapixel

Assume 5 µm gaps, 10 µm x 10 µm plate, 20 V.

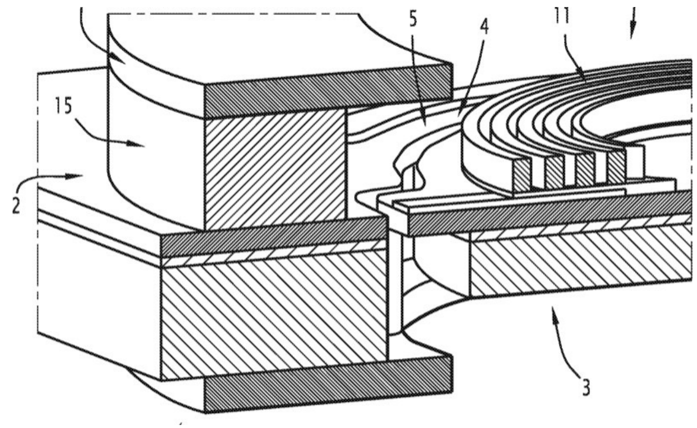
Control: individual pixel (DRAM)

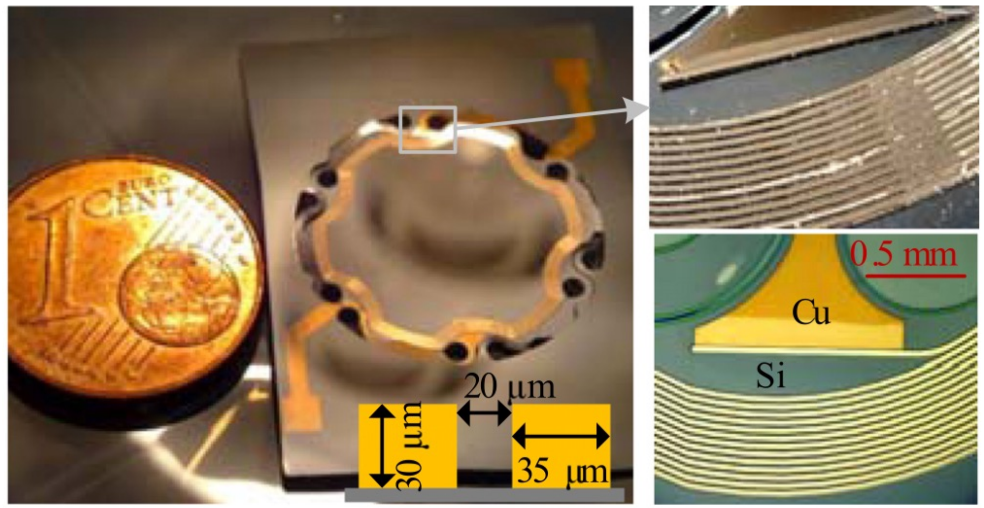
Comparable total force density…

Electromagnetic MEMS loudspeaker
(this paper explains well some design trade-offs)



15 mm diameter





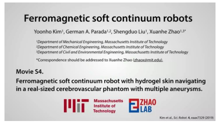
Efficiency (ratio of acoustic power to electrical power) is 10^{-5} , ie 0.001%
Compared to existing loudspeakers in mobile applications, authors claim similar efficiency

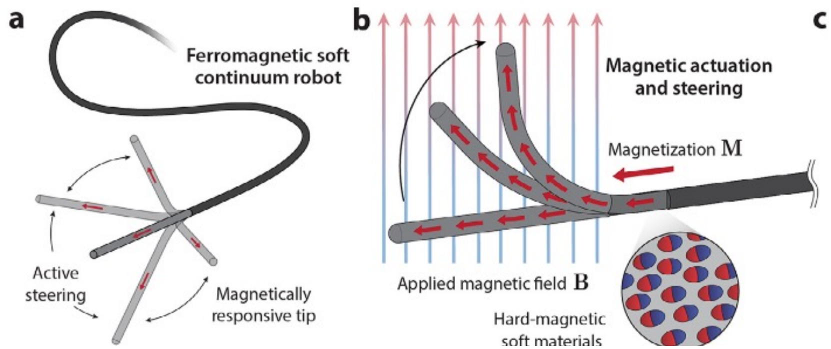
Typical home loudspeaker efficiency 1%.

E Sturtzer, et al. «High Fidelity MEMS Electro- dynamic Micro-Speaker Characterization. », Journal of Applied Physics, (2013), 214905. https://doi.org/10.1063/1.4808334

Forces on magnets in the human body

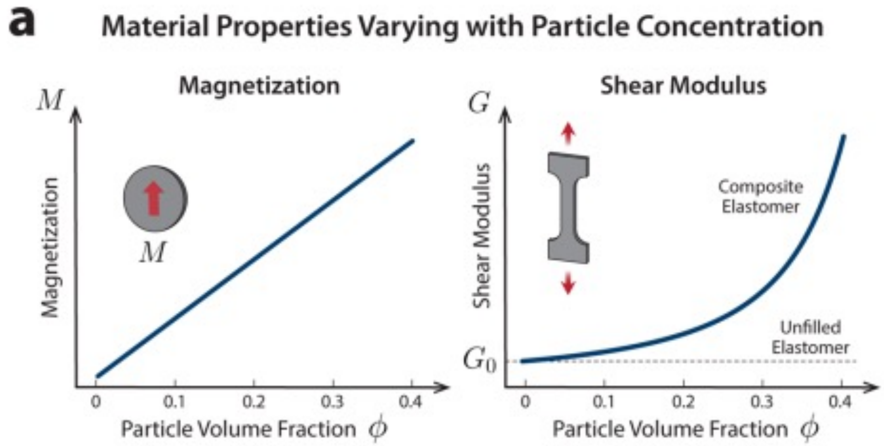
1 .Guided soft EM robots (flexible magnets):





Soft “hard” ferromagnetic robots: Soft polymer (eg silicone) + hard ferromagnetic filler powder

- Assuming: uniform filler dispersion & magnetization
- Assuming: actuation magnetic field << demagnetizing field
- Assuming: $\mu_0 \approx \mu_{air} \approx \mu_{silicone} \approx \mu_{filler}$



The magnetization M is the average magnetic moment density in a medium of volume V, and is proportional to the filler volume ϕ and individual particle magnetization M_p

$$M = \phi M_{\mathrm{p}}$$

As the filler volume ϕ increases the robot becomes more rigid as modeled by the Mooney model

$$G = G_0 \exp\left(\frac{2.5\phi}{1 - 1.35\phi}\right)$$

G the shear modulus
G_0 the unfilled polymer modulus

The robot is modelled as a deformable “hard” magnet and is subject to both forces and torques

$$F_m = m\nabla\vec{B}$$

$$\tau_m = m\times \vec{B}$$

- Kim, Y, and X Zhao. "Magnetic soft materials and robots." *Chemical Reviews* 122.5 (2022)
- Kim, Y., Parada, G. A., Liu, S. & Zhao, X. “Ferromagnetic soft continuum robots.” *Science Robotics* **4**, eaax7329 (2019).

<u>Forces on magnets in the body</u>

1. Guided soft robots (flexible magnets):

Trade off: <u>Softness</u> vs <u>Magnetization strength</u>

Higher particle fill factor: stiffer (less bending), but higher magnetic force
There is an optimum volume fraction for actuations

Model of a simple beam under magnetic field
l is the beam length
D the beam diameter

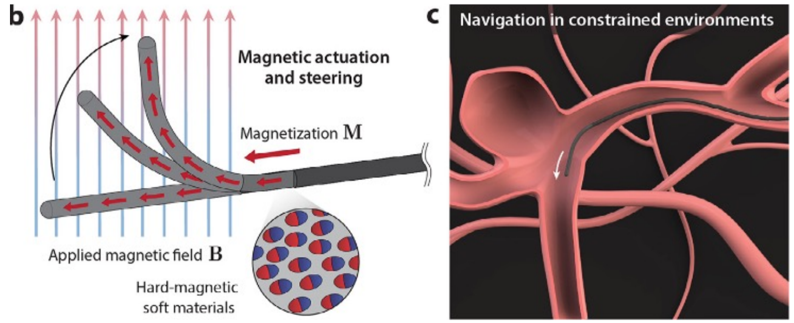
$\left(\frac{l}{D}\right)^2$ is a scalar based on the beam geometry.
Longer and slender beam are easier to deform

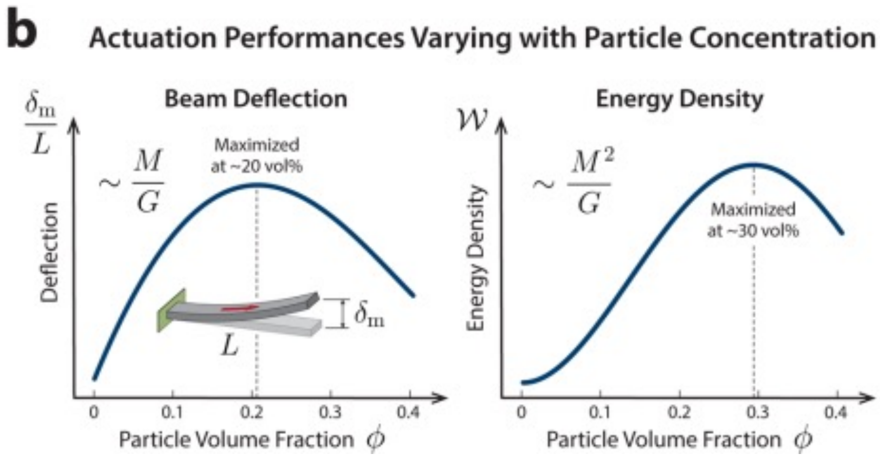
The beam's **normalized deflection** and **energy density** (at small deflections) depend on the applied magnetic field B, the magnetization M, and the shear modulus G

The filler volume to maximize either tip deflection actuation force can be found analytically regardless of beam geometry

$\left(\frac{\delta}{L}\right)_{\max} \Rightarrow \phi \approx 20\%$ $\quad W_{\max} \Rightarrow \phi \approx 30\%$

$\delta \propto L^1$ $\quad W \propto L^0$ (all energy is from outside)





$$\frac{\delta_{max}}{L} \sim \left(\frac{MB}{G}\right)\left(\frac{l}{D}\right)^2$$

normalized deflection of tip of beam

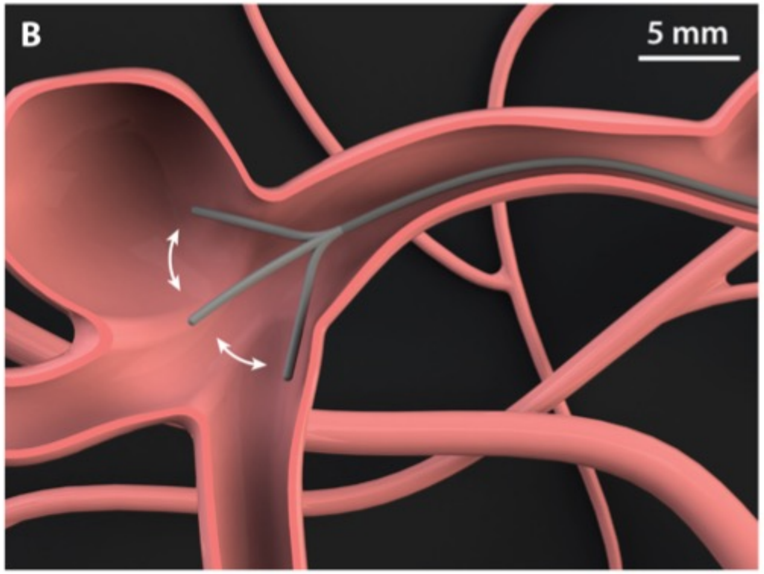
$$W \sim \left(\frac{M^2B^2}{G}\right)\left(\frac{l}{D}\right)^2$$

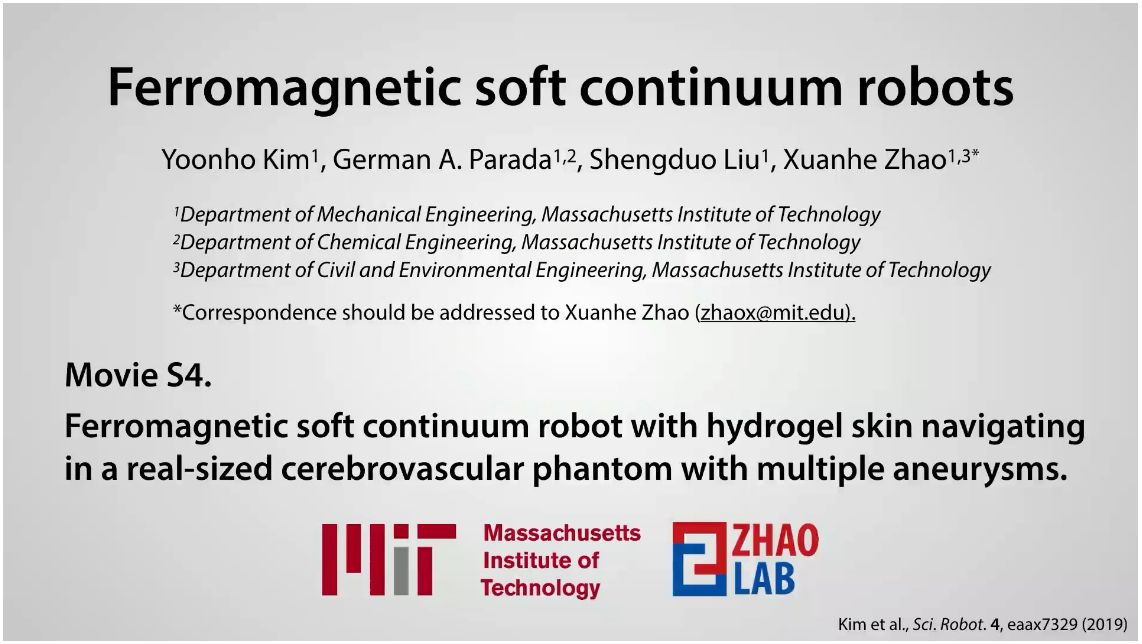
Energy density (for moving tip of beam)

$$\boxed{F = MBA \propto L^2}$$

Kim, Yoonho, and Zhao. "Magnetic soft materials and robots." Chemical Reviews 122.5 (2022): 5317-5364.

Kim, Y., Parada, G. A., Liu, S. & Zhao, X. Ferromagnetic soft continuum robots. *Science Robotics* **4**, eaax7329 (2019).





Forces on magnets in body

2. “Crawling” or undulating soft robots:

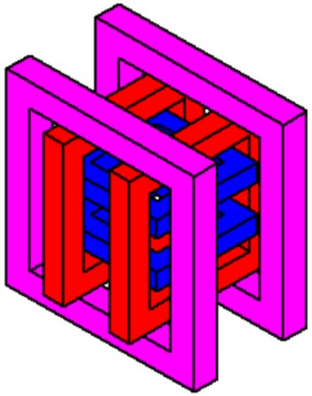
- (S Sakar’s lab, EPFL-STI) : travel within arteries using rotating ***uniform magnetic field*** generated by 3D nested, orthogonal, Helmholtz coils
- Helmholtz coils: $B \sim I/r$ where r is coil’s radius

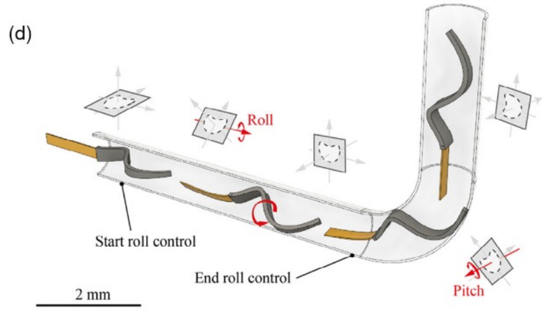
- Single magnetic particle in uniform magnetic field = particle local torque = local stress in the composite
- Continuously rotating magnetic field = undular shape (when confined)
- τ is the torque applied on a single hard ferromagnetic single particle or magnetic moment *m*

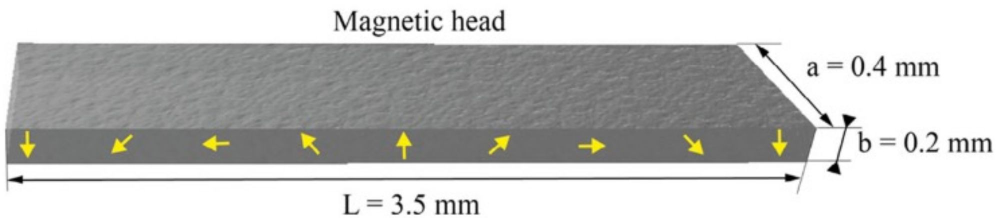
$$\tau_m = m \times \vec{B} \qquad m = VM$$

- Assuming externally cooled coils in Helmholtz setup, current density is intensive: $I \sim L^2$

$$B \propto L \quad \text{and} \quad \tau \propto L^4$$







- Pancaldi, L. *et al.* Locomotion of Sensor-Integrated Soft Robotic Devices Inside Sub-Millimeter Arteries with Impaired Flow Conditions. *Advanced Intelligent Systems* **4**, 2100247 (2022).
- Abbott, Jake J. “Parametric Design of Tri-Axial Nested Helmholtz Coils.” Review of Scientific Instruments 86, no. 5 (May 2015): 054701.https://doi.org/10.1063/1.4919400.

Forces on magnets in body

- Medical setups can *justifiably* use lots of energy to generate actuation to travel within the human body

- Right (S. Sakar's lab, EPFL-STI-IGM) : travel within arteries using rotating *uniform magnetic field* generated by 3D nested, orthogonal, Helmholtz coils

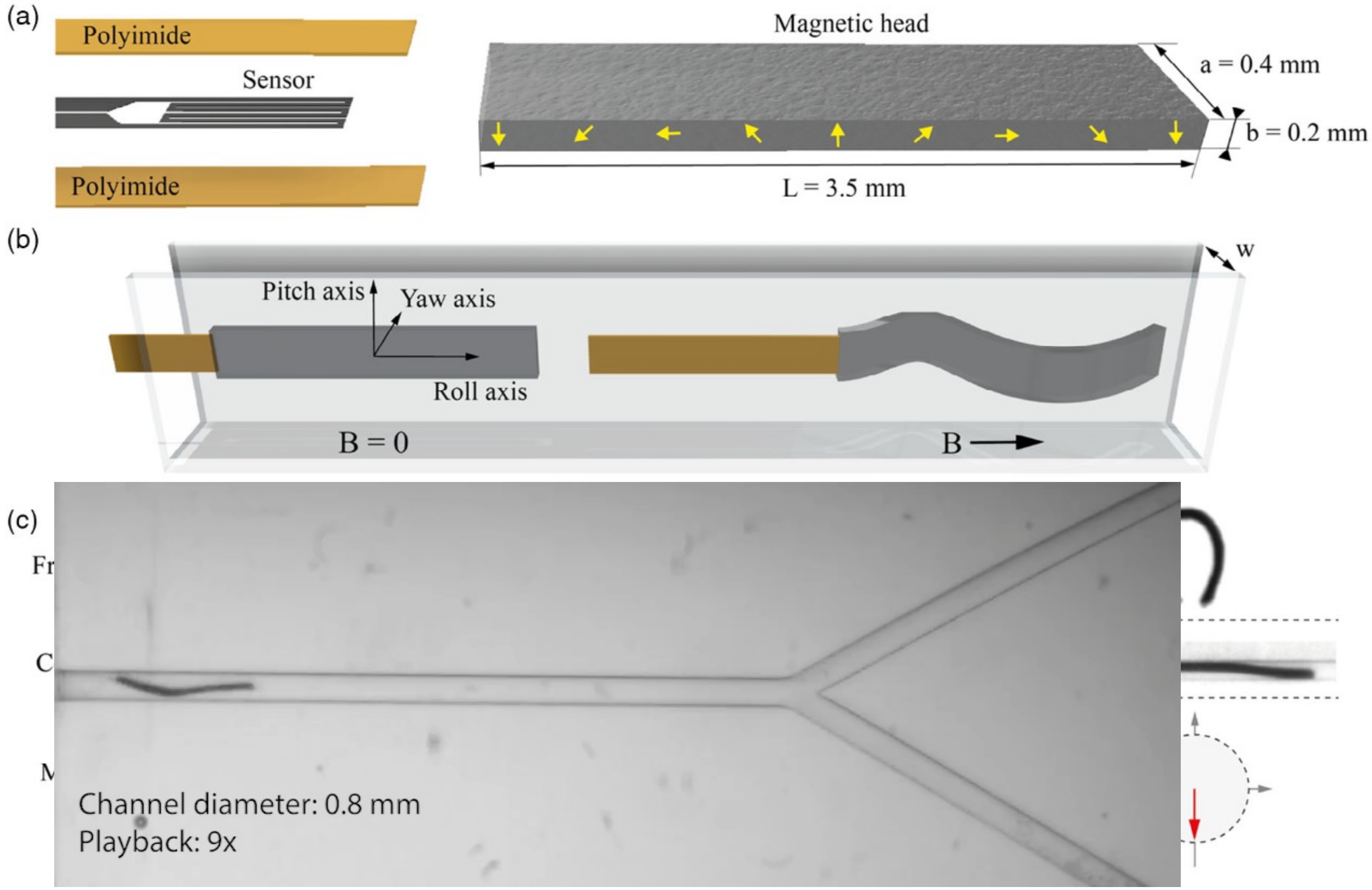


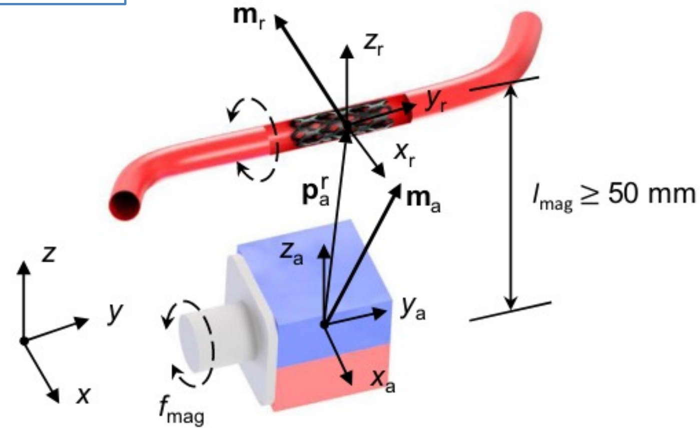
Figure 1. Development of the sensor-integrated soft robotic device. a) Illustration showing the individual components of the device; the polymer tether (250 µm × 3 µm cross-section) with embedded flow sensor and the ferromagnetic head. Yellow arrows indicate the alignment of the magnetic dipoles. b) (Left) Illustration showing the roll, pitch, and yaw axes of the device inside a rectangular channel. (Right) Upon application of the actuation field, B_N, the head bends and contacts the channel walls. c) The shape of an untethered magnetic head along with the shape the same head takes when it is attached to the tether and inserted into a 0.3 mm-wide rectangular channel. The rotating uniform magnetic field is applied on the roll–yaw plane.

Pancaldi, L. *et al.* Locomotion of Sensor-Integrated Soft Robotic Devices Inside Sub-Millimeter Arteries with Impaired Flow Conditions. *Advanced Intelligent Systems* **4**, 2100247 (2022).

Forces on magnets in body

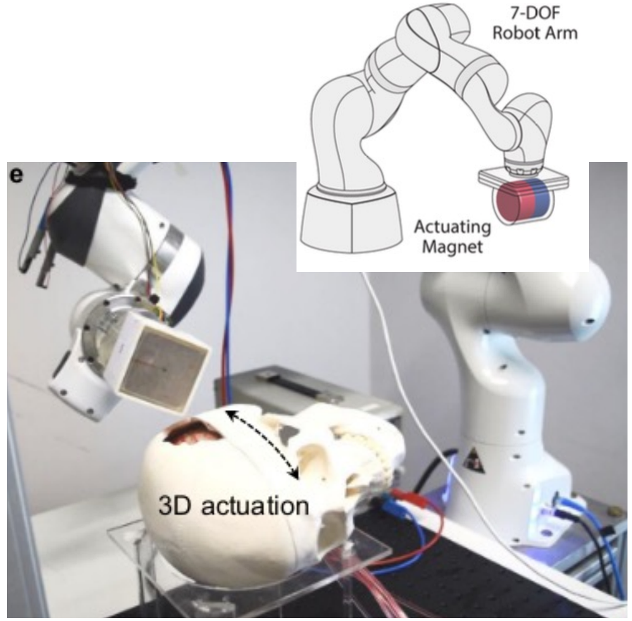
2. “Crawling” or undulating soft robots :

- Another strategy to get local shape changes and force on the magneto-elastomer is to move and rotate large permanent magnets
- Permanent magnet mounted on a robotic arm: *non-uniform magnetic field gradient*
- However (for “large” distance from magnet, i.e., when $L_{mag} >> d_{in\_body\_mag}$):
  - magnetic fields decay as L_{mag}^{-3}
  - magnetic field gradients decay as L_{mag}^{-4}



$$\tau_m = m \times \vec{B} \qquad B \propto L_{mag}^{-3} \qquad \tau_m \propto V L_{mag}^{-3}$$

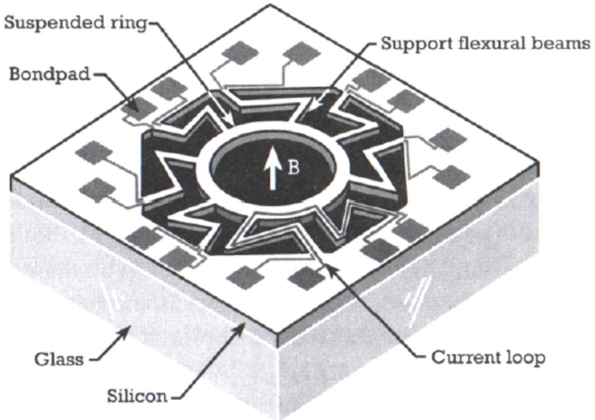
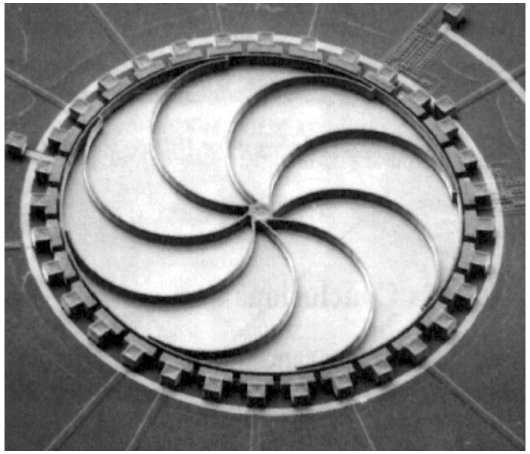
$$m = VM \qquad F_m = m \nabla \vec{B} \qquad \nabla \vec{B} \propto L_{mag}^{-4} \qquad F_m \propto V L_{mag}^{-4}$$

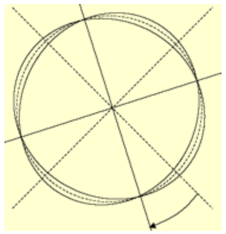


This scaling might seem favorable but only works for larger distances…

- Wang, T. *et al.* Adaptive wireless millirobotic locomotion into distal vasculature. *Nat Commun* **13**, 4465 (2022).
- Kim, Yoonho, and Xuanhe Zhao. "Magnetic soft materials and robots." Chemical Reviews 122.5 (2022): 5317-5364.
- Nelson, Bradley et al . “Microrobots for Minimally Invasive Medicine.” Annual Review of Biomedical Engineering 12, (2010): 55–85.

Ring Gyro: Comparison between Electromagnetic and Electrostatic actuation and sensing

Electromagnetic transducer :	*Electrostatic transducer for comparison :*
	



deformation of the ring $x_1 = 0.149 F_1 \dfrac{R^3}{E.I} = 0.149 F_1 \dfrac{12R^3}{E.w \cdot h^3} = \dfrac{F_1}{k}$

$$k = \frac{E.w}{1.8}\left(\frac{h}{R}\right)^3 = \frac{190 \cdot 10^9 . 10^{-5}}{1.8}\left(\frac{10}{1000}\right)^3 = 1Nm \quad \text{(+ add suspensions)}$$

M. W. Putty and K. Najafi, "A micromachined vibrating ring gyroscope," in *Proc. Digest, Solid-State Sensors and Actuators Workshop*, Hilton Head, SC, June 1994, pp. 213–220. (Nickel gyro)

F. Ayazi, et a, Journal Of Microelectromechanical Systems, 2001
"A HARPSS Polysilicon Vibrating Ring Gyroscope", l. (poly-Si gyro)

Ring Gyro: Comparison between Electromagnetic and Electrostatic actuation and sensing

<table>
<tr>
<td>Actuation :

$F_{em} = I \cdot l \cdot B$

Num: l=100µm, I=100µA, B=0.1T
$F_{em} = 10^{-4} \cdot 10^{-4} \cdot 0.1 = 1nN$
so approx 1 nm motion (static)</td>
<td>$F_{es} = \dfrac{\varepsilon_0 A V^2}{2d^2}$

Num: l=10µm, w=10µm, d=2µm, V=3V
$F_{es} = \dfrac{8.82 \cdot 10^{-12} \cdot 10^{-5} \cdot 10^{-5} \cdot 9}{2 \cdot 4 \cdot 10^{-12}} = 1nN$</td>
</tr>
<tr>
<td colspan="2">Coriolis (orthogonal spring model) : excite 2nd mode (at 45°)
$x_1 = Q \cdot F_1 / k$ $\quad x_2 \cong 2\Omega Q x_1 / \omega_0$ $\quad v_2 = \omega_0 x_2 \cong 2\Omega Q x_1$
Num : Q=1000, F=1nN, k=1Nm, $\omega_0 = 10^5$, Ω=1°/s
x_1=1µm $\quad x_2$=0.02µm $\quad v_2$=4mm/s</td>
</tr>
<tr>
<td>Sensing:
EMF $\quad F_B = qv \cdot B$ $\quad$ and
$qE_{ind} = qvB$
=> $\quad V_{ind} = B \cdot l \cdot v_2 = B \cdot l \cdot \omega_0 \cdot x_2$

Num : $V_{ind} = 0.1 \cdot 10^{-4} \cdot 4 \cdot 10^{-3} = 4 \cdot 10^{-8} V$
Low impedance (R) = easy to amplify</td>
<td>$V_{em} = V_0 \cdot \dfrac{\Delta C}{C_0} \dfrac{C_0}{C_{par}} \cong V_0 \cdot \dfrac{x_2}{d} \dfrac{C_0}{C_{par}}$

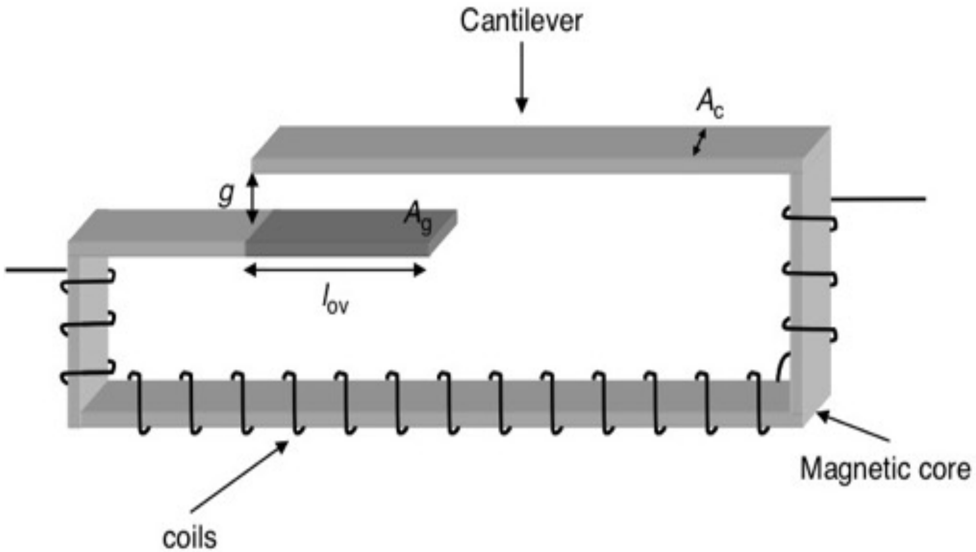
Num : $C_0 = 0.4 fF \quad C_{par} \cong 2pF$
$V_{ind} = 3 \cdot 0.02 \cdot 0.4 \cdot 10^{-3} / 2 = 10^{-5} V$
High impedance (C)</td>
</tr>
</table>

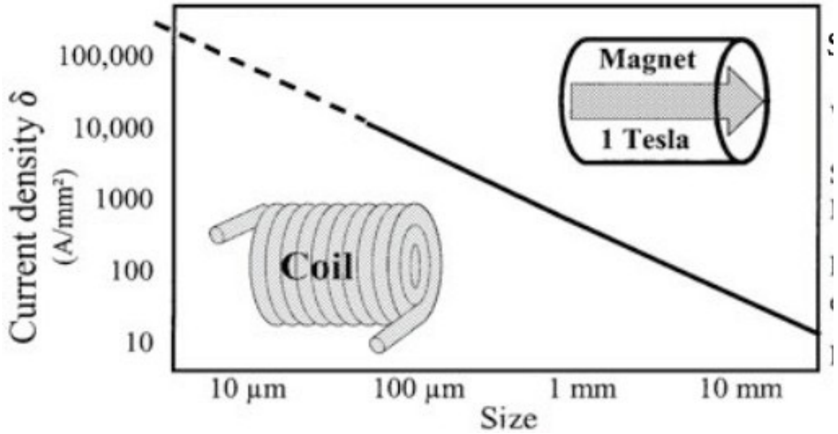
All above number are very approximate. This is to give an input for comparison of transducers principles. In reality, the well-designed electromagnetic device has similar performance as the electrostatic device because B field is externally produced.

EM5: Electromagnetic motors

(greatly simplified)

Electromagnetic motors with variable gap





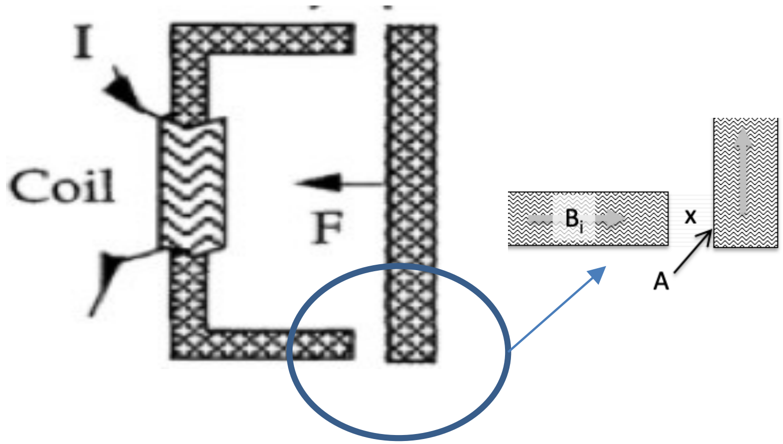
Current density to reach 1 T

<u>Our approximations</u>:

- Variable gap: the mechanical energy that can be extracted is simply given by the difference of magnetic energy (ON/OFF state) times the volume displaced. Neglect fringing
- The reluctance of the core is neglected (e.g. iron core)

See Electromagnetic actuators, in "Scaling Issues and Design of MEMS", S. Baglio, S. Castorina and N. Savalli (2008) Wiley

EM 5.1 Non-polarized **EM actuators** *(= only coil and soft ferromagnetic core)*



$$B_i = \mu\mu_0 \frac{NI}{l}$$

I: current in coil
N: number of windings in coil
l: length of coil
μ: permeability of core

Magnetic energy density in the gap: $w_m = \frac{1}{2\mu_0} B_i^2$ $\quad [w_m] = J/m^3 = N/m^2$ $\quad$ gap= x area= A

When the current *I* is switched from OFF to ON, the increase in magnetic energy density in the gap is:

$$\Delta w_m = \frac{1}{2}\mu_0 \mu^2 \frac{N^2 I^2}{l^2}$$

Mechanical work of the actuator, for a displacement x in the gap *(neglecting the change of reluctance)*

$$\Delta E_{mec} = \Delta w_m A\, x$$

Scaling rule: we assume homothetic scaling, N=constant

Non-polarized **EM actuators** ***(= only coil and soft ferromagnetic core)***

$$w_m = \frac{1}{2\mu_0} B_i^2$$

<u>**Case 5.1 a)**</u> <u>no current limitation due to heating</u>, in motors with a soft ferromagnetic core, the maximum energy density is limited by **saturation field of the magnetic core**:

$B_{max} \approx$ 1 Tesla

The magnetic energy density in the gap is then: $\Delta w_{m,sat} = cst \cong 10^6 \; J/m^3 \quad = 10\, bar$

Power output: $\boxed{P_{mec} = f_0 \cdot \Delta w_{m,sat} \cdot A \cdot x \quad \propto \quad f_0 \cdot L^3}$ *f_0: actuation frequency*

If we assume $f_0 \propto L^{-1}$ $\quad P_{mec} \propto L^2$

<u>Dissipated power</u>:

Current needed to reach saturation (1 T)

$$\Delta w_{m,sat} = \frac{1}{2}\mu_0 \mu^2 \frac{N^2 I_{sat}^2}{l^2} \qquad I_{sat} = \sqrt{\Delta w_{m,sat} 2 \frac{l^2}{\mu_0 \mu^2 N^2}} \propto L$$

Power dissipated in the coil : $P_{dis} = RI^2 \propto L^{-1} \cdot L^2 \propto L$

Scaling of power ratio, or the **efficacy** : $\boxed{\frac{P_{mec}}{P_{dis}} \propto L}$

Smaller = less efficient

Non-polarized EM actuators ***(= only coil and soft ferromagnetic core)***

Case 5.1b) scaling down coil size: the **the coils can no longer carry enough current to reach saturation** of the magnetic core.

Constant current density in order to keep Joule heating limit (ie, to not melt the wires)

$$B_{coil} = \mu \frac{\mu_0 N I}{l} \propto \frac{L^2}{L} \propto L \qquad \Delta w_m = \frac{1}{2}\mu_0 \frac{\mu^2 N^2 I^2}{l^2} \propto L^2$$

Power output:

$$\boxed{P_{mec} = f_0 \cdot \Delta w_m \cdot A \cdot x \quad \propto \quad f_0 \cdot L^5}$$

f_0: actuation frequency

=> *mechanical power drops very quickly when downsizing !*

Dissipated power in the coil:

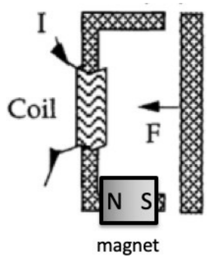
$$P_{dis} = R I^2 \ \propto L^3$$

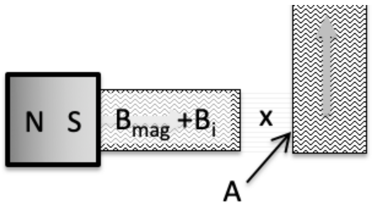
Power ratio

$$\boxed{\frac{P_{mec}}{P_{dis}} \quad \propto L^2}$$

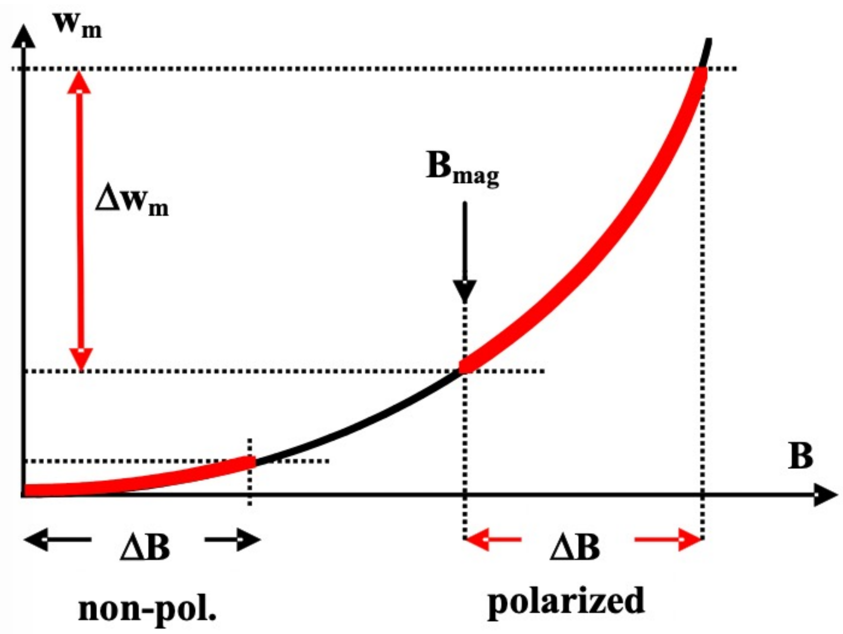
When miniaturized, because of **thermal dissipation limitation**, the current in the coil cannot bring the core to its maximal energy density (saturation) => the actuator is sub-optimal

EM 5.2 Polarized EM actuators (with a permanent magnet in the core)





Since the magnetic energy scales as B^2, a given field added by the coil is produces more magnetic energy in the gap when biased by a permanent magnet



Total magnetic field of magnet + coil :

$$B_{tot} = B_{mag} + B_{coil}$$

Magnetic energy is proportional to B^2 :

$$w_m = \frac{1}{2\mu_0} B_i^{\,2}$$

$$B_{tot}^2 \approx B_{mag}^2 + 2B_{mag}B_{coil} \qquad For\ B_{mag} >> B_{coil}$$

$$\Delta w_m = w_{mag} + \frac{1}{\mu_0} B_{mag} B_{coil}$$

Polarized ***EM actuators (with a permanent magnet in the core)***

Gain in magnetic energy density in the gap when switching ON the current:

$$\Delta w_m = \frac{1}{2\mu_0} 2 B_{mag} B_{coil} = \mu B_{mag} \frac{NI}{l}$$

Scaling of energy density:

$$\Delta w_m \propto L^{-1} I \propto L$$

Power ouput

$$P_{mec} = f_0 \cdot \Delta w_m A x \propto f_0 \cdot L^4$$

$$P_{mec} \propto L^3$$

=> less bad than unpolarized, here power scales with volume

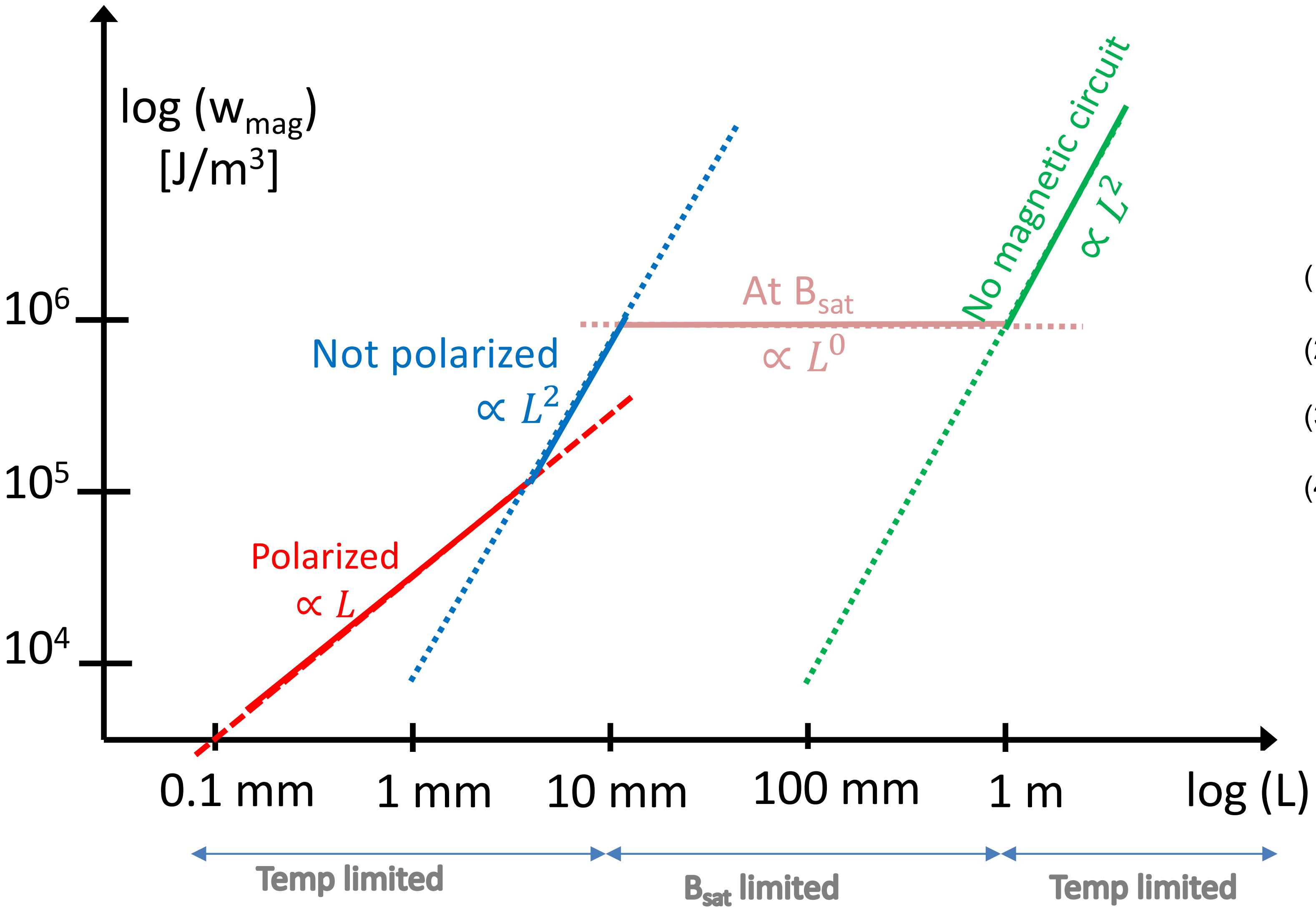
Power ratio

$$\boxed{\frac{P_{mec}}{P_{dis}} = \frac{f_0 \Delta w_m A x}{R I^2} = \frac{L L^3}{L^{-1} L^4} \propto f_0 \cdot L}$$

- The model is over-simplified (just a rough idea about scaling)
- Manufacturing limitation on coils with many turns

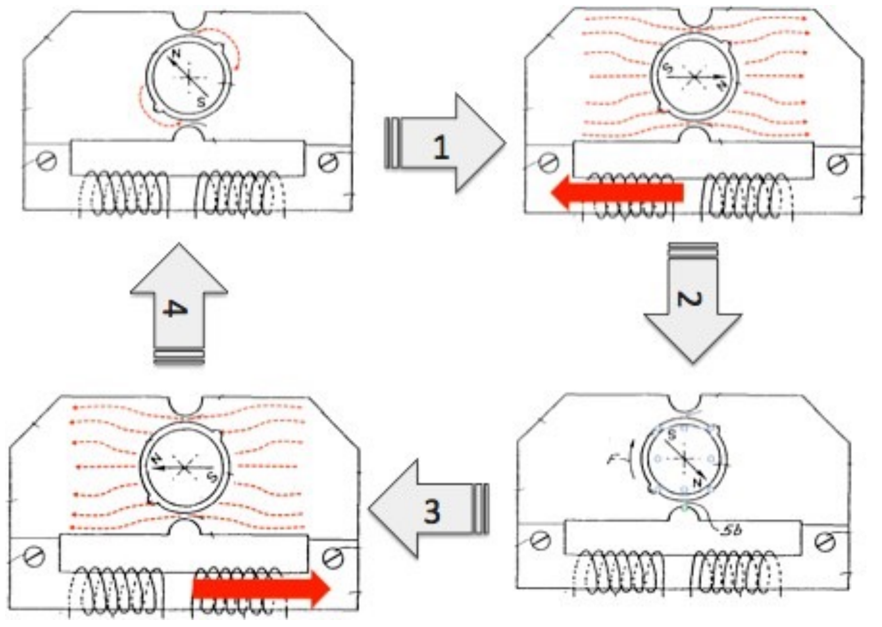
Summary of Scaling of power density in electromagnetic actuators.

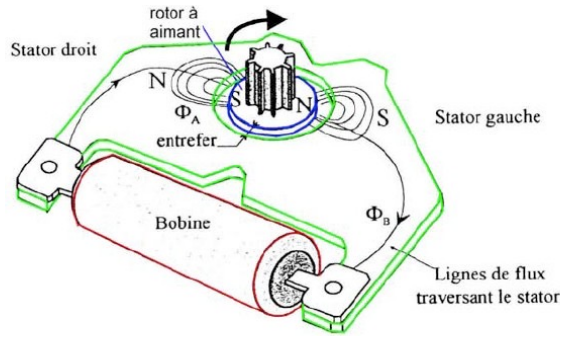
Important !

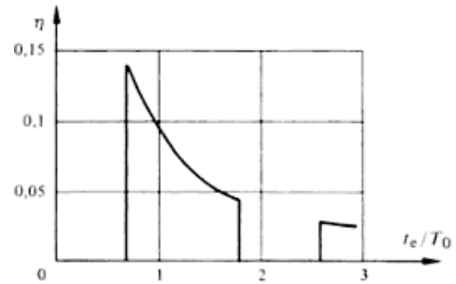


(1) polarized, thermally limited (~L) at very small scale,
(2) not polarized, thermally limited (~L^2) at small scale,
(3) B_{sat} limitation at medium scale,
(4) no magnetic circuit, thermally limited (~L^2) at very large scale

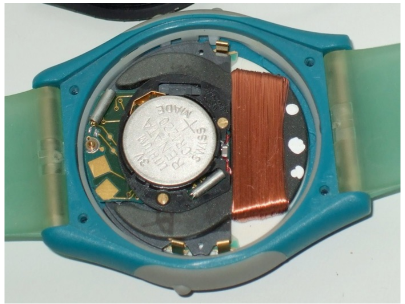
Lavet Stepper motor







- Watch motor (Lavet type) have very good efficiency (60%).
- Coil is big compared to the rotor (which is the permanent magnet)



Quartz watch (approximate) power budget:

- 1/3 to drive the quartz oscillator
- 1/3 to move needles
- 1/3 battery self-discharge